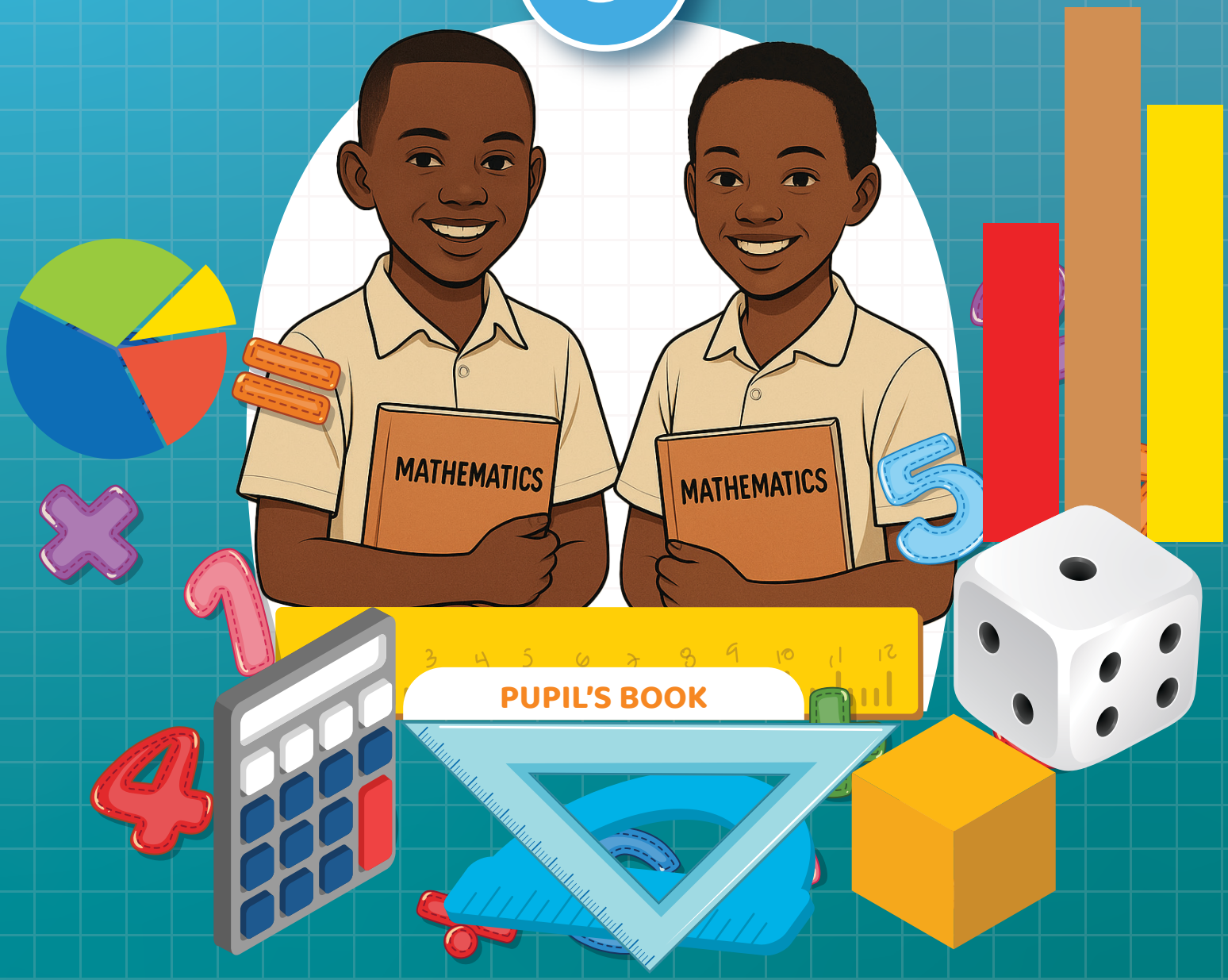
Mathematics

PRIMARY 6

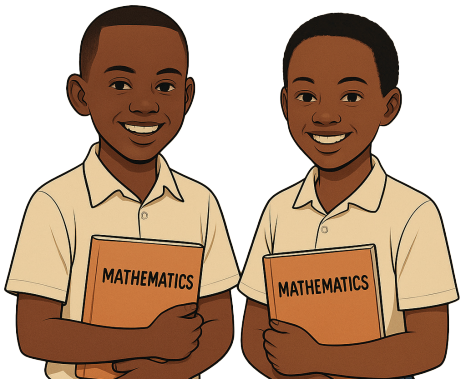


PUPIL'S BOOK

MATHEMATICS

Primary 6

PUPIL'S BOOK



Version Edited in 2025

FOREWORD

Dear Pupil,

The Rwanda Basic Education Board (REB) is honored to present the Mathematics books for Primary Six. This Textbook is designed to support the effective implementation of the Competence-Based Curriculum (CBC); ensuring learner-centered teaching practices that align with Rwanda's vision of empowering learners to reach their full potential, actively participate in society, and pursue meaningful careers.

As Rwanda advances toward building a knowledge-based economy, education plays a pivotal role. Teachers are central to this transformation, guiding learners to connect classroom knowledge with real-life experiences, thus fostering individual growth and national development.

This student book serves as a comprehensive resource for teaching Mathematics at **Primary Six**. The subject is an integral part of the CBC, aimed at equipping learners with essential knowledge, skills, values, and attitudes necessary for becoming responsible and competent citizens. Through this subject, learners are encouraged to develop problem-solving abilities and contribute positively to their communities.

In pursuit of quality education, the Government of Rwanda emphasizes the importance of using teaching and learning materials that are aligned with the curriculum.

Although this guide provides answers to all pupil activities, teachers are encouraged to independently work through the exercises to better evaluate and understand learners' responses.

I sincerely thank everyone who contributed to the refinement of this textbook. Special recognition goes to the REB team who coordinated the entire process. We thank all partners, teachers, and individuals who were involved in the process from start to the end. Every feedback was highly valued and helped to improve learning outcomes of our students.

Dr. MBARUSHIMANA Nelson
Director General, REB

ACKNOWLEDGEMENT

I would like to extend my sincere appreciation to everyone who contributed to the refinement of the Mathematics book for Primary Six. This achievement would not have been possible without the active involvement of various stakeholders.

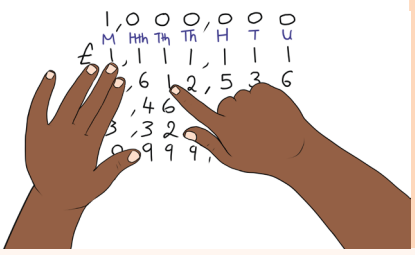
Special thanks go to Rwanda Basic Education Board (REB) staff; subject experts; university lecturers and teachers for their valuable work in the writing of this textbook.

I am also grateful to all our education development partners whose support was instrumental throughout the process.

MURUNGI Joan,
Head of Curriculum,
Teaching and Learning
Resources Department/REB

TABLE OF CONTENT

UNIT 1



READING, WRITING, COMPARING AND CALCULATING WHOLE NUMBERS BEYOND 1,000,000

Key unit Competence: You will be able to read, write, compare and make calculations on whole numbers beyond 1 000 000.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding

- Identify the place values of digits beyond 1,000,000.
- Explain the concept of adding a 7 digit number by a 7 digit number which involves carrying.
- Explain the concept of subtracting larger numbers with borrowing.
- Explain the concept of multiplying larger numbers.
- Explain the concept of dividing a 7 digit number by a 3 digit number.
- Describe the steps taken when rounding off numbers.

Skills

- Read and write numbers correctly in figures or in words.
- Compare numbers using >, < or =
- Calculate numbers involving addition, subtraction, multiplication and division.
- Round off numbers.
- Solve problems involving addition, subtraction, multiplication and division.

Attitudes and values

- Appreciate the importance of accuracy in reading and writing numbers and assessing how big they are.
- Respect others when working on in group to solve a mathematical problem.

1.0. Introduction

In this unit, you will strengthen your understanding of large numbers and learn how to work with them confidently. You will practice reading and writing numbers in words and figures, compare numbers using symbols, and perform calculations involving addition, subtraction, multiplication, and division.

Mastering these skills will help you solve real-life problems, such as handling large quantities, understanding population figures and managing money.

Introductory Activity

Given the 6 number cards with the following digits: 2, 8, 0, 4, 6 and 5.

a) Form the different numbers and tell the place values for the given digits.
b) Read the numbers formed based on the place values.
c) Compare the numbers formed.
d) Add the pair of numbers you formed and read the answer.
e) Try to multiply each number by 50.

- How many digits are in the answer?
- Can you read the number obtained?
- Can you divide each number by 5?
- Where do we use numbers in our real life?

1.1. Place value and value of a digit for a whole number

Activity 1.1

Look at the number cards:

7 8 3 0 5 9 1

- Form a number without changing the position of digits.
- Draw a place value table in your exercise book.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		7	8	9	0	5	9	1

- Use the place value table to fill in the digits of the number 7,830,591 in the correct place values.
- Write down the place value of each digit from the table.
- In which areas of your life can you apply place values?

Summary

1. The place value of a digit tells us how much it is worth.
2. The place value depends on the position of the digit in the number.
3. To find the place values, draw a table.
4. Put the digits in the table, starting from the right side (ones).
5. To find the value of a digit, multiply it by its place value.

Example 1

What is the place value of each digit in 8,356,421?

Solution

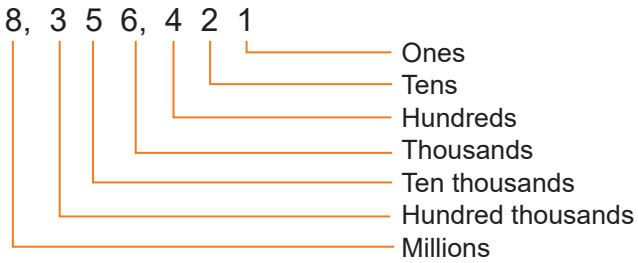
Method 1

Count the digits in the number. There are 7 digits, now draw a place value table as shown below:

Period	**Millions**			**Thousands**			**Units**		
Place value	H	T	O	H	T	O	H	T	O
Number			8	3	5	6	4	2	1
			8 ×1,000,000	3 × 100,000	5 × 10,000	6 × 1,000	4 × 100	2 × 10	1 × 1
Value			8,000,000	300,000	50,000	6,000	400	20	1

- The place value of 8 is millions.
- The place value of 3 is hundred thousands.
- The place value of 5 is ten thousands.
- The place value of 6 is thousands.
- The place value of 4 is hundreds.
- The place value of 2 is tens.
- The place value of 1 is ones.

Method



Example 2

What is the value of 8 in 4,835,634?

Solution

Period	**Millions**			**Thousands**			**Units**		
Place value	H	T	O	H	T	O	H	T	O
Number			4	8	3	5	6	3	4
			4×1,000,000	8×100,000	3× 10,000	5×1,000	6×100	3×10	4×1
Value			4,000,000	800,000	30,000	5,000	600	30	4

Therefore, the value of 8 is 800,000.

Application activity 1.1

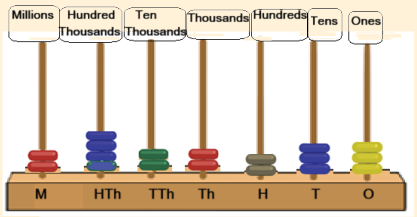
1. Write the place value of each digit in the following:
   (a) 2,312,983 (b) 7,676,405 (c) 10,101,899 (d) 853,925,732
2. Write the place value of the underlined digits in the following:
   (a) 3,459,874 (b) 356,295,712 c) 42,356,890 (d) 42,239,098
3. Write the value of each digit in the following:
   (a) 24,567,400 (b) 208,567,120 (c) 120,394,456
4. Write the value of the underlined digits in the following:
   (a) 34,475,776 (b) 687,034,230
5. Describe how you can identify the value of 6 in 4,567,890.

1.2. Forming, writing and reading numbers beyond 1,000,000

Activity 1.2

Given more number cards with digits from 0 to 9 placed in a bowl.

- Pick **7 random digits** from the bowl
- Arrange the digits in a meaningful order to create your own unique **7-digit number.** For example: 2,422,233
- Represent the number on abacus, starting from ones.
- Read the number and write it in figures.



Summary

When writing whole number in figures:

- First, group the number words in threes starting from the right of the whole number;
- Write the digits of each period in numbers;
- Read and write each period separately;
- Use a comma to separate millions from thousands and units;
- Add the values for getting the right figure.

Example 1: Use number cards, abacus or a table, make and write one, million, two hundred seventy thousand, one hundred thirty-six in figures.

Solution

One million ----------1,000,000

Two hundred seventy thousand ---------- 270,000

One hundred thirty-six ---------- 136

1,270,136

Example 2: A Refugee Camp received three hundred forty-two million, six hundred two thousand, six hundred thirty-one Rwandan francs for buying food for the refugees. Place the number on a place value chart and write the amount in figures.

Solution

Three hundred forty-two million ---------- 342,000,000

Six hundred two thousand ---------- 602,000

Six hundred thirty-one ----------631

342,602,631

Application activity 1.2

1. Write the following numbers in figures:
   b) Fifteen million, three hundred fifty-six thousand, four hundred thirteen.
   c) Eighty-three million, sixty- six thousand, two hundred thirty.
2. A school collected four hundred fifty-six million, five hundred forty-five thousand, two hundred Rwandan francs as school fees. Write this amount in figures.
3. Five hundred twelve million, five hundred forty-nine thousand Rwandan francs was spent by a school in a year. Write the amount in figures.
4. Nineteen million Rwandan francs was spent by a company to print books. Write the amount in figures.

1.3. Reading and writing numbers beyond 1,000,000 in words

Activity 1.3

Match the number in figures to their corresponding number in words.

2,999,999	Fourteen million, one hundred forty thousand, two hundred nineteen.
1,259,000	Two million, nine hundred ninety - nine thousand nine hundred ninety-nine.
14,140,219	One million, two hundred fifty - nine thousand

a) Explain your working steps when matching the numbers.
b) The population of the country is 12,279,742 in words. Use the same steps to write this number in words.
c) Why is it necessary to know how to write numbers in words?

Summary

When you add 1 to 999,999, you get 1,000,000. To write numbers in words:

- First, group the given digits in threes starting from the right to the left side of the whole number;
- Each group of three digits is known as a period;
- Draw a place value table and then write each digit under its place value;

Write the value of each period in words. Then write the names of the periods to separate one period from another;

- Use a comma to separate millions from thousands and thousands from units;
- The periods that are millions, thousands and units are written without 's', means that there is no plural form.

Example 1

A country has a population of 5,600,002. Write the population in words.

Solution

- Group the population in groups of three digits (Hundreds, Tens, Ones).
- Draw a place value table and fill in the digits.
- Write in words the values in the three-digit groups below.

Millions			Thousands			Units		
		O	H	T	O	H	T	O
		5	6	0	0	0	0	2
Five			Six hundred			two		

The population is five million, six hundred thousand and two.

Example 2

A water tank holds 82,999,555 litres of water. Write the litres in words.

Solution

Millions			**Thousands**			**Units**		
H	T	O	H	T	O	H	T	O
	8	2	9	9	9	5	5	5
Eighty-two			Nine hundred ninety-nine			Five hundred fifty-five		

82,999,555 litres: Eighty-two million nine hundred ninety-nine thousand five hundred fifty-five litres.

Application activity 1.3

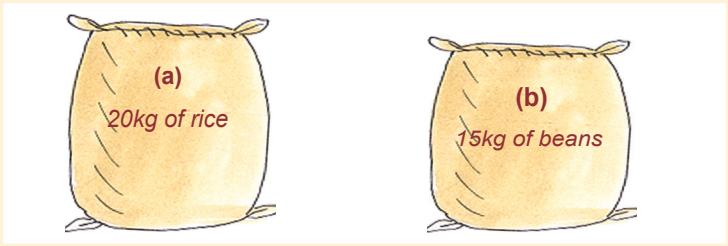
1. Write the following whole numbers in words:
   (a) 10, 406, 078 (b) 9, 700, 956 (c) 721, 569, 216

2. Kalisa bought a cow at 456,700 Frw. Write the amount in words and show your working steps
3. Abeli collected 5,417,257 litres of milk from his farm in five months. Write the number of litres that he collected in words.
4. Agatha deposited 4,565,090 Frw in the bank. Write the amount that she deposited in words. Is it necessary to be able to write numbers in words?

1.4. Comparing numbers using <, > or =

Activity 1.4

Study the figure below and answer the questions that follow.



a) Convert 20 kg and 15 kg in grams.
b) Which one is bigger than the other?
c) Complete the sentence using either greater than, smaller than or equal to
d) The Sack (a) is _____Sack (b)

Summary

To compare numbers

- Compare them referring to their place values.
- Check the biggest number against the smallest number.
- You can compare numbers by counting the number of their digits.
- For numbers with the same number of digits, compare the digits with the same place value from left to the right, the digit with the highest value indicates the greatest number.

Example 11

Compare 6,312,542 and 6,312,452 using $<, >$ or $=$

Solution

Draw a place value table and fill in the numbers.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		6	3	1	2	5	4	2
		6	3	1	2	4	5	2

Compare the digits in each place value from left to right.

6 = 6, 3 = 3, 1 = 1, 2 = 2, 5 > 4 in the hundreds of units place value.

Therefore, 6,312,542 > 6,312,452.

Example 2

Imanirere sold clothes worth 2,560,320Frw in 2015. She sold clothes worth 4,576,670 Frw in 2016. Compare the sales over the two years using $<, >$ or $=$

Solution

Draw a place value table and fill in the numbers.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		2	5	6	0	3	2	0
		4	5	7	6	6	7	0

Compare the digits in each place value from left to right.

2< 4 in the millions place value. Therefore 2,560,320 < 4,576,670.

Application activity 1.4

1. Use <, > or = to compare the following:
   (a) 260,340_____60,430,730.
   (b) 8,855,631____8,855,136
   (c) 302,831,547 _____30,283,154.
2. Camille harvested 5,562 tonnes of beans and Kamanzi harvested 5,256 tonnes of beans. Who harvested more beans?
3. Mukagasana deposited 2,506,590 Frw in the bank. Her son deposited 259,000 Frw. Who deposited more money?
4. Hospital A admitted 45,679 patients in 2016 while hospital B admitted 67,890 patients in the same year. Which hospital admitted more patients? Explain your answer.
5. The district A collected 4,853,825 Frw in taxes while the district B collected 4,197,900 Frw. Which district collected more money?

1.5. Arranging numbers in ascending and descending order

Activity 1.5

Use the number cards. Four pupils play the game on a line.

942,704 942,407 1,496,066 1,496,606

- Order number cards in ascending order. One pupil puts a card on the line
- Write the order of numbers on slips of paper.
- Change the order and arrange the number cards in descending order.
- Write down the new order.
- What do you consider when ordering the number cards both in ascending and descending? Explain your procedures.

Summary

- Ascending order is the arrangement of numbers from the smallest to the biggest number while descending order is the arrangement from the biggest to the smallest number.
- If two numbers have the same number of digits, a greater number is the one with a greater digit in the same place value and vice- versa.
- A number with more digits is bigger than the number with fewer digits.

Example

Arrange the following numbers in both ascending and descending order.

1,707,055 1,770,550 3,025,446 3,205,446

Solution

Use a place value table to compare the digits and arrange numbers in ascending or descending orders.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		1	7	0	7	0	5	5
		1	7	7	0	5	5	0
		3	0	2	5	4	4	6
		3	2	0	5	4	4	6

Use a place value table to compare and arrange the numbers in ascending or descending order.

Start comparing from the highest place value to the lowest place value.

- In ones of millions, 1 = 1, 3 = 3, but 1 < 3 or 3 > 1.
- In hundred thousands, 7 = 7 and 0 < 2,

- In thousands, 0 < 7 and 0 < 2,
  so, 3,205,446 > 3,025,446; 3,025,446 > 1,770,550; 1,770,550 > 1,707,055.
- Also, 1,707,055 < 1,770,550; 1,770,550 < 3,025,446 and 3,025,446 < 3,205,444.
- Numbers arranged in ascending order: 1,707,055; 1,770,550; 3,025,446; 3,205,446.
- Numbers arranged in descending order: 3,205,446; 3,025,446; 1,770,550; 1,707,055.

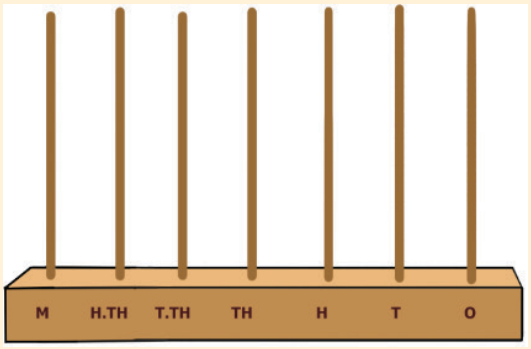
Application activity 1.5

1. Order the following numbers in ascending order.
   a) 1,673,421; 1,065,345; 1,671,241; 1,065,234
2. Order the following numbers in descending order.
   a) 4,963,427; 4,427,963; 4,369,427; 4,724,963

1.6. Addition of whole numbers beyond 1,000,000 using both table and wooden vertical abacus

Activity 1.6

Given two numbers: 2,034,315 and 3,432,541.



- Represent the two numbers on abacus using beads and respecting the place values of digits.
- Place the first number 2,034,315 on the abacus and then put the second number 3,432,541 on the abacus respecting the place values.
- After putting the 2 numbers on the abacus, count the number of beads on each place value.
- Write the obtained number represented by the combined beads;
- Use the obtained number to complete:
- 2,034,315 + 3,432,541 = ...
- Can you work out: 2,034,315 + 3,432,541 in another way? Compare the answers.

Summary

When adding big numbers, like 1,000,000 or more, follow these steps:

- Write them in a column, making sure the digits are in the right place (ones, tens, hundreds, etc.).
- Add the digits in the ones place first, then move to the left. That is starting from right to left, from ones, to tens, thousands, ten thousands, hundred thousands and millions.

- If a place value adds up to 10 or more, carry the extra number to the next place value.
- Keep adding until you reach the leftmost digits.
- The key words used for addition include total, sum, altogether, combined.

Example

Find the sum of 4,629,208; 2,823,004 and 5,987,253.

Solution

Arrange the digits according to place values. Then add the two whole numbers starting from the right (ones) to the left.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		$^{2}4$	$^{1}6$	$^{1}2$	9	2	$^{1}0$	8
		2	8	2	3	0	0	4
	+	5	9	8	7	2	5	3
	1	3	4	3	9	4	6	5

Application activity 1.6

Add the following numbers using abacus or the place value table.

a) 4,985,670 + 2,322,502 =

b) 6,232,343 + 2,432,234 + 1,067,103 =

c) 9,088,033 + 9,000,046 =

d) 1,602,444 + 2,622,433 + 5,789,987 =

1.7. Solving real life problems involving addition of numbers beyond 1,000,000

Activity 1.7

In a country, there are 12,000,000 males and 3,000,000 females.

a) What should you do to know the total number of people in that country?

b) Compare the number of males to that of females.

c) Give examples of where addition is applied in your daily life.

Summary

To solve a word problem involving addition:

- First, read and interpret the question correctly;
- when adding whole numbers, arrange the digits in the table according to the place values;
- If the numbers do not have the same number of digits, use zeros to act as place holders to ensure proper alignment of each digit according to place values;
- Start adding from right to the left, that is from ones to tens, thousands, ten thousands, hundred thousands and millions;
- The answer which is determined from addition is called "the Sum or the total".

Example

Builders used 5,762,426 bricks to build the foundation of a house and 3,028,987 bricks to put up walls of the house. Find the total number of bricks that were used to complete the house.

Solution

Arrange the digits according to place values. Then add the two whole numbers starting from the right (ones) to the left (millions).

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		5	7	$^{1}6$	$^{1}2$	$^{1}4$	$^{1}2$	6
	+	3	0	2	8	9	8	7
		8	7	9	1	4	1	3

The total bricks that were used to complete the house is 8,791,413.

Application activity 1.7

1. A dairy cooperative sold 1,123,456 and 8,467,619 litres of milk on Monday and Tuesday respectively. How much milk was sold in the two days?
2. Publishing companies A, B and C supplied the following number of textbooks to primary schools in the same district last month. A supplied 1,345,346 copies, B supplied 1,206,460 copies and C supplied 1,600,400 copies. What is the total number of books supplied by all three publishing companies?
3. Kayitesi sold 1,625,255 kg of maize flour in the first year and 3,268,450 kg of maize flour in the second year. What was her total sales in the two years?

1.8. Subtraction of numbers beyond 1,000,000 using both table and wooden vertical abacus

Activity 1.8

Mukangarambe went to the shop with 29,680 Frw, she spent 17,240 Frw to buy materials.

- How much money remained?
- What is the most suitable method can you use to work out the difference?

Summary

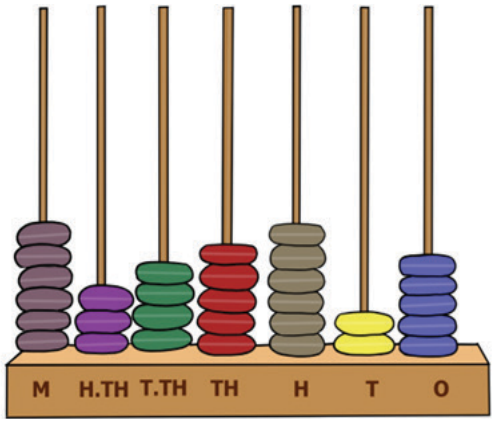
- When subtracting large numbers, arrange the numbers in vertical order, placing each digit in its correct place value.
- When subtracting a bigger digit from a smaller one in the same place value, borrow 1 ten (10) from the digit in the next place value to the left. Add it to the smaller digit on your right being subtracted from. This is called borrowing and re- grouping. Then subtract.
- The answer we get from subtraction is called "the difference."

Example

Subtract: 6,345,625 - 2,124,304= .....

Solution

Method 1: subtract 6,345,625 - 2,124,304 using abacus

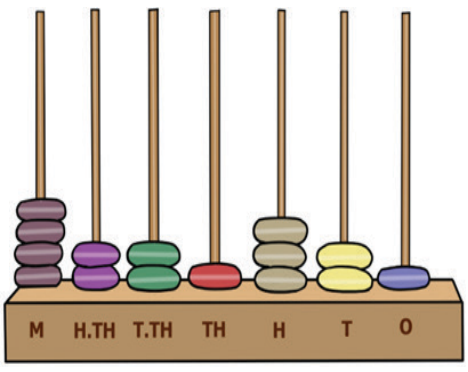


Since we have 6,345,625 - 2,124,304, we place the beads representing the first number 6,345,625.

- Then, put 5 beads on the spike of ones,
- Put 2 beads on the spike of tens, put 6 beads on the spike of hundred,
- Put 5 beads on the spike of thousands,
- Put 4 beads on the spike of ten thousand,
- Put 3 beads on the spike of hundred thousand
- Put 6 beads on the spike of millions.

6 ,3 4 5, 6 2 5 is the number represented on the abacus.

- To subtract 2,124,304, we take away beads that represent this number.
- Take away 4 beads from the spike of ones.
- Take away 0 beads from the spike of tens.
- Take away 3 beads from the spike of hundred.
- Take away 4 beads from the spike of thousands.
- Take away 2 beads from the spike of ten thousand.



- Take away 1 bead from the spike of hundred thousand
- Take away 2 beads on the spike of millions.
- The beads that remain on the abacus represent the number 4,221,321.
- Therefore, 6,345,625 - 2,124,304 = 4,221,321.

Method 2: subtract 6,345,625 - 2,124,304 using place value table

- Arrange the digits according to their place values.
- Put the larger number at the top of the table followed by the smaller number. Subtract the two whole numbers starting from the right to the left.
- Remember to borrow then re-group where necessary as you subtract.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		6	3	4	5	6	2	5
	-	2	1	2	4	3	0	4
		4	2	2	1	3	2	1

Therefore, 6,345,625 - 2,124,304 = 4,221,321

Application activity 1.8

1. Subtract the following numbers using abacus or place value table
   a) 6,000,101 – 4,999,011 =
   b) 6,291,569 – 4,687,263 =
   c) Take away 1,039,042 kg from 3,642,110 kg
   d) 9,008,200 Frw – 8,000,200 Frw =
2. What is the difference between 6,326,428 books and 8,040,249 books?

1.9. Solving real life problems involving subtraction of numbers beyond 1,000,000

Activity 1.9

Gabiro borrowed 5,345,600 Frw from the bank. He has so far paid 3,000,560 Frw.

1. Determine the remaining amount of money that Gabiro must pay the bank.
2. Give examples where subtraction is applied in your daily life.

Summary

A real life problem involving the subtraction has some key words such as: Difference, Less than, subtracted from, take away, minus, decrease, fewer, remaining, etc.

To solve such a problem:

- First read and interpret the question correctly.
- When subtracting whole numbers, arrange the digits in the table according to the place values. Make sure the large number on top of the small number.
- If numbers do not have the same number of digits, use zeros to act as place holders to ensure proper alignment of each digit according to the place values.
- Start subtracting from the right (ones) to the left side. Make sure you borrow and re-group where you find that the top digit is smaller than the bottom digit.
- When faced with a problem involving both addition and subtraction, always carry out addition first and subtract last.

Example

A juice company produced 7,003,453 litres last week. It sold only 5,654,000 litres in the week. How many litres of juice remained unsold?

Solution

- Arrange the digits in the table according to the place values.
- Put the large number at the top of the small number.
- Subtract the two whole numbers starting from the right to the left. Remember to borrow then re-group where necessary

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O
		$^{6}\cancel{7}$	$^{9}\cancel{0}$	$^{9}\cancel{0}$	$^{1}3$	4	5	3
	-	5	6	5	4	0	0	0
		1	**3**	**4**	**9**	**4**	**5**	**3**

The remained unsold 1,349,453 litres of juice.

Application activity 1.8

1. What is the difference between 2,798,576 pens and 2,745,568 pens?
2. 3,567,342 babies were born in a country in 2016. If 1,593,599 babies were girls, find the number of boys.
3. There are nine million three hundred twelve thousand six hundred eight animals in the park. Out of these, three million six hundred nine thousand, three hundred twenty-three are zebras. How many animals in the park are not zebras?
4. Gitego harvested twenty-five million, five thousand two hundred fifty kilograms of Irish potatoes. He took away sixteen million, four hundred twenty-eight thousand, five hundred kilograms to sell to the schools. How many kilograms remained?

1.10. Multiplication of numbers beyond 1,000,000 by a 2 or 3-digit number

Activity 1.10

To multiply 1,235,265 by 124, a student used the method described here below. Follow it and provide your comment.

- Multiply the number by 4. Note the answer.
- Multiply the number by 20. Write the answer.
- Now multiply 1,235,263 by 100. Write the answer.
- Add the three answers. Write the answer.

Do you think that there is a mistake? Why did the student multiply by 4, then by 20, then by 100?

Summary

When multiplying a number beyond 1,000,000 by a 3-digit number:

- Multiply each digit of the 3-digit number by the whole number, starting from the units place, and then add the results.
- Remember to place the results in the correct position based on place value and carry over any extra value when needed.

Example: Multiply 1,603,421 by 132.

Solution

Arrange in vertical order according to the place values of each digit.

$$
\begin{array}{rl}
 & 1\,6\,0\,3\,4\,2\,1 \\
\times & \quad 1\,3\,2 \\
\hline
 & 3\,2\,0\,6\,8\,4\,2 \qquad 1{,}603{,}421 \times 2 \text{ (ones)} \\
 & 4\,8\,1\,0\,2\,6\,3\,0 \qquad 1{,}603{,}421 \times 30 \text{ (tens)} \\
+ & 1\,6\,0\,3\,4\,2\,1\,0\,0 \qquad 1{,}603{,}421 \times 100 \text{ (hundreds)} \\
\hline
 & 2\,1\,1,6\,5\,1,5\,7\,2
\end{array}
$$

Therefore, 1,603,421 × 132 = 211,651,572.

Note: It is possible to multiply by 2, then by 3 and then by 1 and consider the appropriate place of the answer.

```
    1 6 0 3 4 2 1
  ×       1 3 2
  ---------------
    3 2 0 6 8 4 2      1,603,421 × 2 ones
  4 8 1 0 2 6 3        1,603,421 × 3 tens
+ 1 6 0 3 4 2 1        1,603,421 × 1 hundred
-----------------------
  2 1 1, 6 5 1, 5 7 2
```

Therefore, 1,603,421 x 132 = 211,651,572.

Application activity 1.10

1. Work out the following:

   a) 986,342 × 76 =

   b) 896,234 × 121 =

   c) 1,112,025 × 111 =

   d) Multiply 2,316,310 by 99.

   e) Multiply 1,076,033 by 104.

1.11. Solving problems involving multiplication of numbers

Activity 1.11

In January, a school admitted 500 learners. Each learner paid 30,000 Frw for school uniform and materials.

- How would you find the total amount paid by all learners?
- Describe the steps you take to get the answer.

Summary

- The problem involving multiplication has one of the following key words: Product, times, multiplied by, each (when referring to repeated groups), every (as in "every group"), double, triple, quadruple (indicating multiple of a number).
- To solve it, read and interpret the question correctly. This will help you to apply the right operation.

Example

A petrol station uses 36 trucks to distribute petrol between its branches in Rwanda. Each truck carries 456,798 litres of petrol. How much petrol was distributed?

Solution

36 trucks each carrying 456,798 litres.

$$\begin{array}{rl} & 456,798 \\ \times & \quad 36 \\ \hline & 2,740,788 \quad \text{456,798 × 6 (ones)} \\ + & 13,703,940 \quad \text{456,798 × 30 (tens)} \\ \hline & 16,444,728 \end{array}$$

Therefore 16,444,728 litres were distributed.

Application activity 1.11

1. Mr. Kamanutsi sells 1,200,350 litres of milk per month. How many litres does he sell in a year?
2. A non-government organisation was supposed to deposit five hundred thousand five hundred Rwandan francs as tuition fees for each of the students it sponsors at university. If it sponsors 25 students, how much money should it deposit?
3. Habimana wanted to save tuition fees for her daughter to study at university. At university the tuition fees are 400,000 Frw per term. How much money must he save for her daughter to complete three terms in the year?
4. What is the product of 1,404,055 by 121?

1.12. Division of numbers beyond 1,000,000 by a 2 or 3-digit number

Activity 1.12

Imagine, you are given 1,240,000 Frw to be shared.

- Share it equally among 40 people. How much money does each get?
- Complete your answer: 1,240,000 ÷ 40 = ...
- Explain your working steps.
- Is equal sharing of things / objects/ money with friends a good attitude? Discuss

Summary

When dividing a number beyond 1,000,000 by a 2 or 3-digit number:

- Start with the digits in the highest place value.
- Estimate the nearest number of times a number can be divided.
- Carry the remainder to the next place value if it does not divide exactly.
- Align the digits in order to subtract correctly.

Example 1

Divide: 2,448,768 by 32

Solution:

```
        7 6 5 2 4
32 ) 2 4 4 8 7 6 8
   – 2 2 4                ← 7 x 32
       2 0 8
     – 1 9 2              ← 6 x 32
         1 6 7
       – 1 6 0            ← 5 x 32
             7 6
           – 6 4          ← 2 x 32
             1 2 8
           – 1 2 8        ← 4 x 32
                 0
```

Therefore 2,448,786 ÷ 32 = 76,524

Example 2

Share 1,175,576 kg of beans among 184 parishes.

Solution: Each parish gets 6,389 kg

```
          6 3 8 9
184 ) 1 1 7 5 5 7 6
    – 1 1 0 4             ← 6 x 184
          7 1 5
        – 5 5 2           ← 3 x 184
          1 6 3 7
        – 1 4 7 2         ← 8 x 184
            1 6 5 6
          – 1 6 5 6       ← 9 x 184
                0
```

Therefore, 1,175,576÷184=6,389.

Each parish gets 6,389 kg

Application activity 1.12

1. Work out the following.
   a) 2,026,648 ÷26=
   b) 8,123,518÷34 =
   c) 7,562,296 ÷56 =
2. Share equally 8,164,904 avocado saplings (young trees) among 124 villages.
3. Distribute equally 7,827,831 kg of maize among 333 parishes. How many kilograms does each parish get?

1.13. Solving problems involving division of numbers

Activity 1.13

Get 300 sticks and share them among 10 learners.

a) How many sticks does each learner get?
b) What operation have you used to find the answer?

Summary

When solving a problem:

- First read and interpret the question correctly. This will help you to apply the right operation.
- When dividing, start with the digits in the highest place value.

- Estimate the nearest number of times a number can be divided. If it does not divide exactly, carry the remainder to the next place value.
- Then multiply and subtract.

Application activity 1.13

1. Share 2,026,800 Frw among 24 employees equally. How much does each get?
2. 500 members of the congregation contributed a total of 5,501,000 Frw. How much did each contribute if they paid the same amount.
3. A soda bottling company packed 8,462,376 bottles of soda in crates each containing 24 bottles. Find the number of crates that were packed.

1.14. Rounding off whole numbers to the nearest tens

Activity 1.14

a) Given a number 2,347. Is this number near 2,350 or near 2,340?
b) Given number 8,703, is this number near 8,700 or near 8,710?
c) Explain your working steps.
d) You are rounding off to the nearest tens. What is the importance of rounding off in our daily life? Is it applicable in every situation?

Summary

To round off whole numbers, use the place values and the value of digits.

To round off to the nearest tens:

- First look at the digit in the place value of tens in the whole number.
- If the digit on the right of the required place value is greater or equal to 5, (that is, 5, 6, 7, 8, 9), you round up: Add 1 to the digit in the required place value.
- If the digit on the right of the required place value is less than 5, (that is, 0, 1, 2, 3, 4), you round down: The digit in the required place value doesn't change but all digits to the left change to 0.

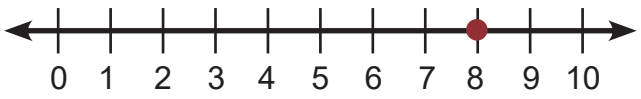
Example 1

Round off 2,458,548 to the nearest tens.

Solution

2, 4 5 8, 5 4 8

Number to the right
Required place value



8 is in the upper limit so it is nearer to 10 than to 0. So, 8 is rounded to 1 ten (10).

$$\begin{array}{r} 2{,}458{,}540 \\ +\quad\quad 10 \\ \hline 2{,}458{,}550 \end{array}$$

Therefore, 2,458,548 rounded to the nearest tens is 2,458,550.

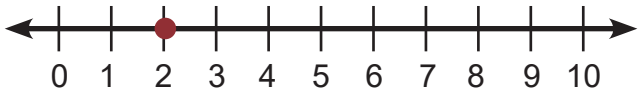
Example 2

Round off 1,602,162 to the nearest tens.

Solution

1, 6 0 2, 1 6 2

Number to the right
Required place value



2 is in the lower limit so it is nearer to 0 than to 10. So 2 is rounded to 0 ten (00)

Therefore, after rounding off 1,602,162 to the nearest tens it becomes 1,602,160

Application activity 1.14

1. Round off to the nearest tens:
   (a) 4,856,796 (b) 6,789,735 (c) 2,234,587 (d) 3,654,857 (e) 62,453,702
2. The average number of goats on a farm is 6,753,927, round off this number to the nearest tens.
3. Find the product of 23,000 and 30. Round off your answer to the nearest tens.
4. A school used 598,992 Frw to buy computers and 100,000 Frw to buy books for its library. How much money did it spend altogether? Round off the answer to the nearest tens. Can the rounded number be accepted in the school report?

1.15. Rounding off whole numbers to the nearest hundreds and thousands

Activity 1.15

- Pick four number cards from the pack as follow: 9,230; 3,290; 219 and 39,289.
- Look carefully at the digits in each particular place value.
- Identify the digits to the right of hundreds place value. Round it up or down.
- Write on the approximate numbers to hundred.
- Now round off 39,289 to the nearest hundreds. Explain your working steps clearly.

Summary

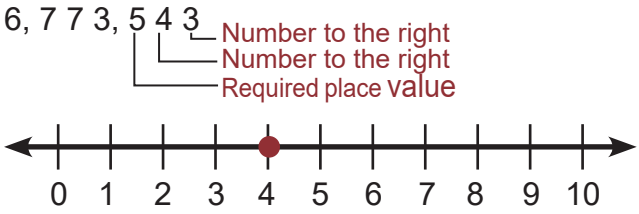
The number on the right hand side of the digit in the desired place value becomes zero.

- To round off whole numbers to the nearest hundreds, consider the number in the tens place value.
- To round off whole numbers to the nearest thousands, consider the number in the hundreds place value.
- If the digit is greater or equal to 5, it is converted to one hundred or thousand then added to the place value of hundreds or thousands according to the required place value.
- If the digit is less than 5, it is converted to zero hundred or zero thousand and then added to the required place value.

Example 1

Round off 6,773,543 to the nearest hundreds.

Solution



6,773,500
\+ 000
6,773,500

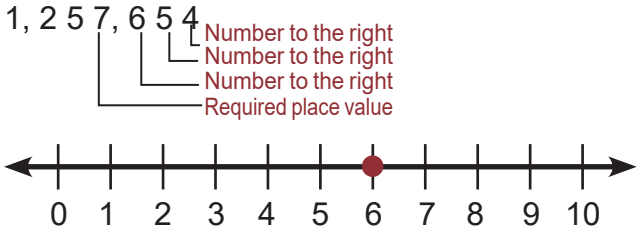
4 is in the lower limit. It is nearer to 0. It is added to 5 to give 5.

Therefore, after rounding off 6,773,543 to the nearest hundreds it becomes 6,773,500.

Example 2

Round off 1,257,654 to nearest thousands.

Solution



1,257,000
\+ 1,000
1,258,000

6 is in the upper limit. It is nearer to 1 thousand, 1 is added to 7 to give 8. Therefore, when 1,257,654 is rounded to the nearest thousands it becomes 1,258,000.

Note: • Rounding off is possible in = estimating materials needed.

- Rounding off is not allowed when you are paying money, at the bank or school fees or in legal calculations = estimating materials needed.

Application activity 1.15

1. Round off the following numbers to the nearest hundreds.
   (a) 3,654,597 (b) 22,987,652 (c) 564,323,940 (d) 3,890,909
2. Round off the following numbers to the nearest thousands.
   (a) 6,068,902 (b) 8,523,174 (c) 64,565,038 (d) 70,309,585

1.16. Rounding off whole numbers to the nearest ten thousands, hundred thousands and millions

Activity 1.16

Study the number cards and answer the following questions:

2,345,789 3,604,800 5,687,231 1,342,798

a) Identify the digits to the right of the ten thousands place value. Round it up or down.
b) Add the rounded digit to the ten thousands place value.
c) Replace all the digits to the right of the ten thousands place value with zeros. What do you notice?

Summary

- To round off to the nearest ten thousands, consider the digit in the thousands place value.
- To round off to the nearest hundred thousands, consider the digit in the ten thousands place value.
- To round off to the nearest millions, consider the digit in the hundred thousands place value.
- If the digit is greater or equal to 5, it is converted to one ten thousand, one hundred thousands, one million respectively, then added to the required place value.
- If the digit is less than 5, it is converted to zero ten thousands, zero hundred thousands, zero million respectively, then added to the required place value.

Example 1

Round off 1,576,798 to the nearest ten thousands.

Solution

1, 5 7 6, 7 9 8
Number to the right
Number to the right
Required place value

1,570,000
+ 10,000
1,580,000

6 is in the upper limit. It is nearer to 1 ten thousands. So, 6 is rounded to 1 ten thousands (10,000).

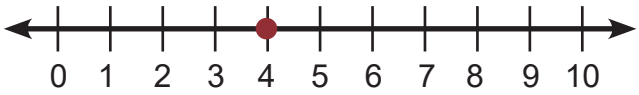
Therefore, when 1,576,798 is rounded to the nearest ten thousands it becomes 1,580,000.

Example 2

Round off 3,540,750 to the nearest hundred thousands.

Solution

3, 5 4 0, 7 5 0
Number to the right
Number to the right
Required place value



3,540,000
+ 000,000
3,500,000

4 is in the lower limit. It is nearer to 000,000. So 4 is rounded to 000,000.

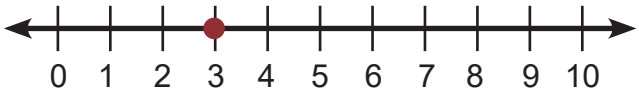
Therefore, 3,540,750 rounded to the nearest hundred thousands is 3,500,000.

Example 3

Round off 7,398,500 to the nearest millions.

Solution

7, 3 9 8, 5 0 0
Number to the right
Number to the right
Required place value



7,000,000
+ 0,000,000
7,000,000

3 is in the lower limit. It is nearer to 0 millions. So, 3 is rounded to 0 million (0,000,000).

Therefore, when 7,398,500 is rounded to the nearest millions it becomes 7,000,000.

Note: In some situations, this is not accepted; to ignore 398,500 is a big loss.

Application activity 1.16

1. Round off the following numbers to the nearest millions:
   (a) 49,546,401 (b) 4 2,560,456 (c) 10,564,670
2. A farmer sold ten of his cows and earned 4,687,300 Frw. Round off his earned money to the nearest ten thousands.
3. Mutesi paid 1,520,500 Frw in tuition fees for her first semester at university. Round off her tuition fees to the nearest hundred thousands.
4. Round off the following numbers to the nearest underlined digit:
   a) 3,120,6<u>4</u>0 (b) 8,670,<u>7</u>98 (c) 7,45<u>6</u>,982 (d) 61,2<u>2</u>3,789 (e) 8,<u>5</u>76,700

1.17. End unit assessment

1.17.1. Questions

Assessing knowledge and understanding

1. Write the place value of each digit?
   (a) 76,767,709 (b) 5,999,999
2. Give the value of underlined digit
   (a) 7,50<u>2</u>,507 (b) 7,<u>8</u>22,034
3. Write 710,023,202 in words
4. Define "place value" and "digit" in your own words.

Assessing skills

5. Compare the following using >, < or =.
   (a) 1,121,277_____1,121,207
   (b) 9,876,534 _____ (3,232,456 + 1,087,653)
   (c) (92,268 ÷ 2) _____(7,689 × 12)
6. Round off to the underlined digits:
   (a) <u>8</u>,765,423 (b) 6,<u>5</u>45,677 (c) 98,7<u>7</u>6,113 (d) 45,36<u>7</u>,089 (e) 9,999,<u>9</u>58
7. A company printed 19,884,345 books last year and 26,326,150 books this year. How many books were printed altogether over the two years?
8. Kamali collected 5,678,950 Frw from milk sales in this month. Write the amount in words.
9. Complete the table below:

Quantity one	Sign	Quantity two	Total
1,456,776 kg	+	2,456,767 kg	
2,555,550 Frw	—		1,365,890 Frw
	×	450	10,688,400 Frw
49,560,000 Frw	÷	7,080 Frw	

10. A company produced 10,964,329 bottles of soda in January, 12,726,455 bottles of soda in February, 18,612,900 bottles of soda in March and 5,046,500 bottles of soda in April.
    a) How many bottles of soda did thecompany produce over the four months?
    b) If the company sold 20,892,600 bottles of soda during those four months, how many bottles of soda remained?

Assessing attitudes and values

11. A bank employee misreads 45,367,089 as 45,637,089. How might this error affect customers? What can be done to prevent such mistakes?
12. Explain to a classmate how you would teach them to round 9,999,958 to the nearest millions. Use clear steps.

1.17.2. Presentation of projects on multiplication and division of numbers using the two-five bead abacus.

UNIT 2

MULTIPLICATION AND DIVISION OF INTEGERS

Key Unit Competence: You will be able to multiply and divide integers.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding

- Describe the steps taken when multiplying and dividing integers.
- Show and explain the concept of integers to solve problems.

Skills

- Apply the concepts of multiplication and division to solve problems involving integers.
- Carry out multiplication and division of integers.
- Explain how integers change in multiplication and division.

Attitudes and values

- Appreciate the importance of accuracy in multiplication and division of integers.
- Respect each other's contribution when working in groups.
- Build confidence when working on integers.

2.0. Introduction

In this unit, you will explore how to multiply and divide positive and negative numbers. You will learn the rules that determine the sign of the product or quotient and apply them to solve problems involving integers.

Understanding these concepts is essential for real-life situations, such as calculating temperature changes, determining gains or losses in finances, and solving more advanced mathematics problems. Through engaging activities, examples, and practice exercises, you will build confidence in working with integers.

Introductory Activity

1. In Mathematics, operations on integers are performed in a range of situations. Suppose you are moving and counting from point A to point B with steps forward for a distance of 20 steps.

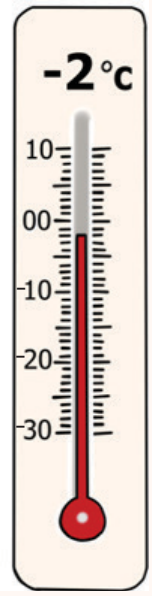
a) What happens if you reach the point B and go back jumping 2 steps four times without changing the direction you are facing? Is the movement positive or negative?

b) How many jumps are needed to cover the distance from point B to A? Give the mathematical operation used for getting the answer.

c) Consider the starting point A as zero. What will happen if you continue jumping and pass to point A four steps?

2. If the thermometer indicates that the temperature of an object is -2 degrees. What is the temperature of another object colder 5 times as the one measured by the thermometer?

3. Give examples on how integers can be used in financial transactions (gains and losses), temperature changes, or in sports scores.



2.1. Multiplying integers using counters

Activity 2.1

- Let us use the counters whose one side is blue and other side is red.
- Consider the red side of the counter as positive (+) and the blue side of the counter as negative (-).

a) Think of multiplying two positive integers using counters, what do you get?

b) Think of multiplying two negative integers using counters, what do you get?

c) Think of multiplying negative and positive integers using counters, what do you get?

A. Multiplying two positive integers

Example 1: Multiply the following: (+3) × (+2) = ...

Solution

- Consider that the multiplication sign stands for grouping. So read (+3) × (+2) as 3 groups of (+2) counters.

- According to our coloured counters, 3 groups of (+2) counters means that there are three groups each group having 2 red counters or 2 positive counters as shown below.

- There are six red counters or 6 positive counters. Therefore (+3) × (+2) = +6

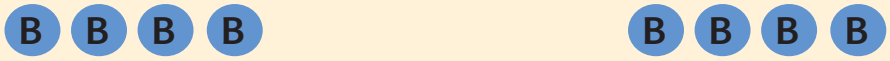
B. Multiplying two negative integers

Example 2: Multiply the following: (– 2) × (– 4) = ...

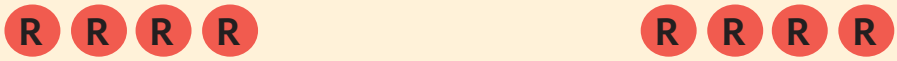
Solution

Note that a negative on the multiplicand is read as opposite. Therefore, (– 2) × (– 4) is read as the opposite of two groups of negative four. But we know that negative four is represented by four blue counters.

- First, make two groups of four counters with blue face showing up and place them on a row.



- Since it is a negative two (-2) which means the opposite two groups, we invert/reverse the blue faces of the counters to have the red faces showing on top.



- Note that, we invert/reverse because of the negative sign on the multiplicand.
- There are eight counters placed with red faces up. Therefore, (-2) x (-4) = +8.

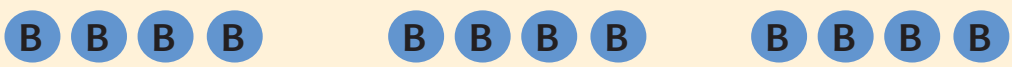
C. Multiplying negative and positive integers

Example 3: Multiply the following (-3) × (+4) = ...

Solution

- Since a negative means the opposite, (-3) × (+4) means opposite of 3 groups of positive four.
- Starting with 3 × (+4) means make three groups with each group having 4 counters placed in a row with red faces facing up as below.

- But the sign negative of (-3) in (-3) × (+4) means the opposite of 3 groups of positive four.
- Therefore, we flip or invert the counters in each case putting the counters with blue faces facing up.



- There are twelve counters placed with a blue face on top. This represents -12.
- Therefore, (-3) × (+4) = -12.

Summary

Multiplying integers using counters is like grouping and combining sets of objects. This method makes multiplying integers more visual and easier to understand.

Using counters helps to see patterns with signs:

- Positive × Positive = Positive
- Negative × Positive = Negative
- Negative × Negative = Positive

For example:

- To multiply (+3) × (+2), think of 3 groups of 2 positive counters each. That makes a total of 6 positive counters.
- To multiply (-3) × (+2), think of 3 groups of 2 negative counters. That makes -6.
- To multiply (-3) × (-2), think of 3 groups of 2 negative counters, but since you are multiplying by a negative, it flips to positive 6.

Application activity 2.1

Use counters to multiply the following:

a) (+4) × (+4) = …… b) (-5) × (-8) = ……..

c) (+2) × (-7) = ……. d) (-3) × (+5) = ……

2.2 Multiplying integers using a number line

Activity 2.2

- Draw a number line on the ground.
- Jump from 0 to 4, then from 4 to 8 and from 8 to 12.
- How many jumps have you made?
- Record your findings.
- What do you notice?

Summary

Multiplying integers using a number line is like making jumps forward or backward.

1. **Positive × Positive:** Move forward or move to the positive side.	Example: (+3) × (+ 2) – Start at 0, – Make 3 jumps of 2 steps forward – You land on 6.
2. **Negative × Positive:** Move backward or move to the negative side.	Example: (-3) × (+2) – Start at 0, – Make 3 jumps of 2 steps backward – You land on – 6.
3. **Negative × Negative:** Move forward or move to the positive side.	Example: (-3) × (-2) – Start at 0, – Normally, -3 × 2 moves backward – But since we multiply by another negative, it flips forward – You land on 6.

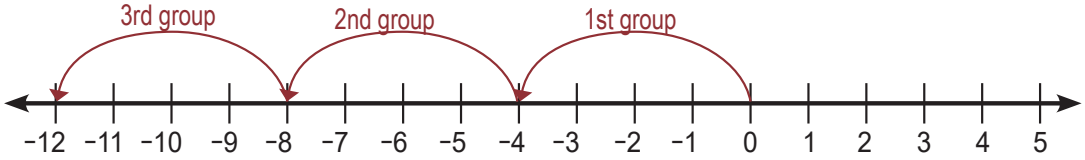
In general, when multiplying integers using a number line, move intervals which are equal to the multiplied number, times the number of the multiplier and remember that multiplication is like repeated addition.

Example 1

Multiply: (+3) × (-4) using a number line:

Solution

- (+3) × (-4) = (-4) + (-4) + (-4)
- (+3)× (-4) means 3 groups of -4 (multiplication is repeated addition)



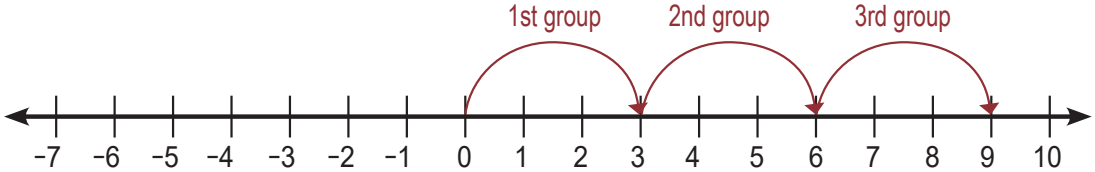
Therefore, (+3) × (-4) = -12

Example 2

Multiply: (+3) × (+3) using a number line:

Solution

- 3 × (+3) = (+3) + (+3) + (+3) (add +3, three times).
- 3 × (+3) = 3 groups of +3



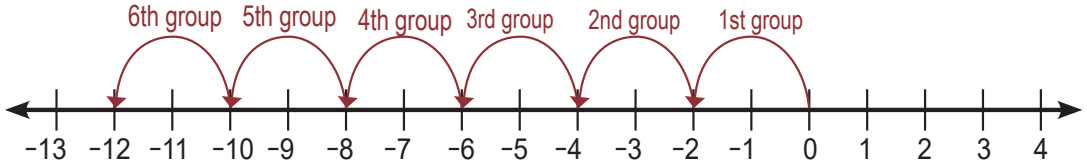
Therefore, 3 × (+3) = +9

Example 3

Multiply (6) × (-2) using a number line:

Solution

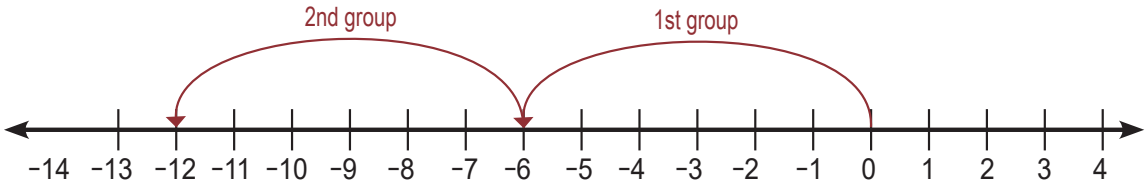
- (+6) × (-2) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) (add -2, six times).
- (+6) × (-2) means 6 groups of -2



Example 4 : (-2) × (+6)

Solution

(-2) × (+6) = opposite of 2 x (+6)



Therefore, (-2) × (+6) = -12

Application activity 2.2

1. Use a number line to multiply the following:
   (a) (+5) × (+8) = (b) (2) × (-11) = (c) (4) × (-6) = (d) (-9) × (3) = (e) (+3) × (12) =
2. Keza makes a stride of 3 gaps on a number line to its right. Find the total integers she makes in 6 strides.
3. Gashumba made 9 jumps. Each jump is represented by 2 gaps on a number line to its left. Write an expression that describes the statement. Find the integer of where Gashumba lands.

2.3. Multiplying integers without using a number line

Activity 2.3

- Given the number cards with numbers 36, -12, 48, and -18
- Pick a number card and match it with the corresponding mathematics statement.
  a) (-3) × (+6) = _____
  b) (+4) × (+9) = ____
  c) (+8) × (+6) = ____
  d) (+2) × (- 6) = _____
- Explain your working steps to get to the answer.

36

–12

48

–18

Summary

Multiplying integers is easy when you remember a simple rule about signs.

1. Multiply the numbers normally (ignore the signs for now).
2. Look at the signs and use the rule:
   - Positive × Positive = Positive
   - Negative × Positive = Negative
   - Negative × Negative = Positive

Example: Multiply the following without using a number line

(a) +7 × (-9) = (b) -3 × (+12) = (c) (+5) × (+8) = (d) (-7) × (-8) =

Solution

(a) (+7) × (-9)= -63 (b) (-3) × (+12) = -36 (c) (+5) × (+8) =+40 (d) (-7) × (-8) = +56

Application activity 2.3

Work out the following:

(a) +4 × (+4) = (b) (-5) × (-8) = c) (+1) × (-7) =

(d) -11 × (+9) = (e) -125 × 12 =

2.4. Dividing integers using a number line

Activity 2.4

- Draw a number line on the ground.
- Jump from +28 to +21, then from +21 to +14, then from +14 to +7 and finally from +7 to 0.
- How many jumps have you made?
- Record your findings and discuss.
- What do you notice?

Summary

Dividing integers using a number line is like counting how many times one number fits into another.

- Division is like repeated subtraction.
- Start from zero and make intervals equivalent to the divisor in length until you reach the dividend.
- The number of intervals becomes the answer.
- If the dividend is in the positive direction (quotient), the integer is positive. Example:

6 ÷ 2, start at 0, make jumps of 2 until you reach 6, you land on 6 after 3 jumps and the answer is 3

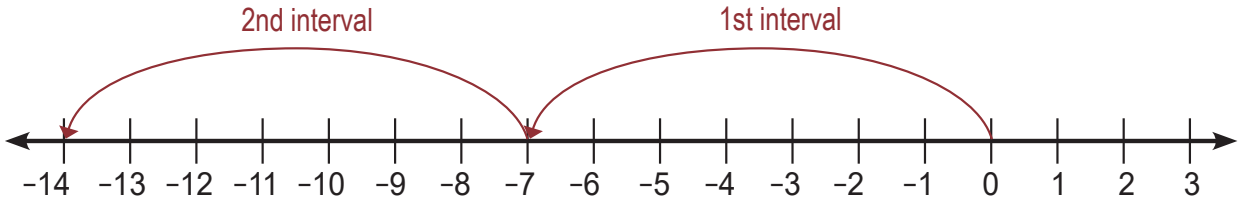
- If the dividend is in the negative direction, the integer is negative. Example: -6 ÷ 2, start at 0, make jumps of 2 backward until you reach -6, you land on -6 after 3 jumps and the answer is -3.

Example 1

Divide: (-14) ÷ (+7) =

Solution

Start moving from 0 making intervals of 7 steps up to -14.



The number of intervals is 2 in the negative direction.

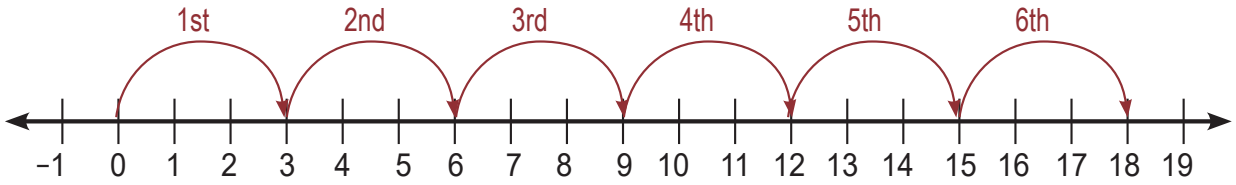
Therefore, (-14) ÷ (+7) = -2

Example 2

Divide (+18) by (+3)

Solution

Start moving from 0 making intervals of 3 steps up to +18.



The number of intervals is 6 in the positive direction.

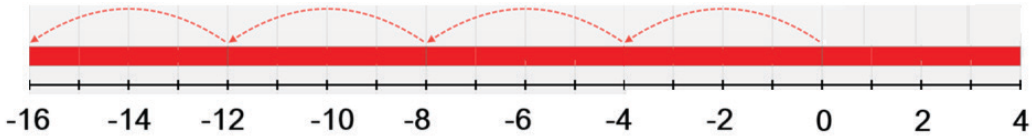
Therefore, (+18) ÷ (+3) = +6

Example 3:

Divide (-16) ÷ (-4)

Solution

- Start moving from 0 making intervals of 4 steps up to -16.



- Applying the sign rule and since both numbers are negative, the answer is positive. Therefore, (-16) ÷ (-4) = +4 , it means 4 number of jumps in the same direction.

Application activity 2.4

By using a number line, divide the following:

(a) (+20) ÷ 5 = (b) (-16)÷(+2) = (c) (+18)÷(-3) = (d) (-28)÷(+4)=

(e) (+12) ÷ (-3) = (f) (+27)÷(+3) = (g) (-9) ÷3= (h) (-24)÷8=

2.5. Dividing integers without using a number line

Activity 2.5

Pick number cards that complete the following statements:

(a) (+18) ÷ (-3) = (b) (+77) ÷ (+7) =

(c) (-50) ÷ (10) = (d) (+12) ÷ (-3) =

−5 −4 −6 +11

– Explain the steps taken to get the answer.

Summary

Dividing integers is simple when you follow the rules for signs.

1. Divide the numbers normally (ignore the signs for now).
2. Check the signs and follow this rule:
   - Positive ÷ Positive = Positive. Example: 8 ÷ 2 = 4
   - Negative ÷ Positive = Negative. Example: (-8) ÷ 2 = -4
   - Negative ÷ Negative = Positive. Example: (-8) ÷ (-2) = 4

In general,

3. Dividing integers with the same sign gives a positive. $(+)\div(+)=+$ or $(-)\div(-)=+$
4. Dividing integers with different signs gives a negative. $(+)\div(-)=-$ or $(-)\div(+)=-$

Example

Divide the following without using a number line.

(a) +12 ÷ (-3) = (b) -21 ÷(+7) = (c) -48 ÷(- 8)= (d) +8 ÷(+8) =

Solution

(a) +12 ÷ (-3) = -4 (b) -21 ÷ (+7) = -3 (c) -48 ÷ (-8) =+6 (d) +8 ÷(+8) = +1

Application activity 2.5

Divide the following without using a number line.

(a) (+39) ÷ (–13) = (b) (–21) ÷ (–7) = (c) (–50) ÷ (+10) =

(d) (+44) ÷ (+11) = (e) (–18) ÷ (–6) =

2.6. Solving problems involving multiplication and division of integers

Activity 2.6

- Think of a number.
- Divide it by +4.
- Multiply the answer by –2.
- If the final answer is 6 what is the number?
- Explain your working steps.

Summary

Integers help us understand many things in real life. They help us describe changes in different situations.

- **Describing loss and profit:** Example: If a shop makes 10,000 Frw; we say a propfit of 10.000 Frw or + 10,000. If a shop loses 5,000 Frw; we say a loss of 5,000Frw of -5,000.
- **Temperature rise and fall:** Example: If the temperature goes up 5 degrees, we say +5. If it goes down 7 degrees, we say -7.
- **Ascending and descending altitude:** Example: A plane flying 1,000 meters up is +1000. A submarine 200 meters below sea level is -200.

Example

Temperatures of an area increases by +4^{o}C per hour during the day. What is the total temperature increased in 6 hours?

Solution

- 1 hr = +4^{o}C
- 6hrs = +4^{o}C x (+6) = +24^{o}C

Therefore, the total temperature increase in 6 hours is +24^{o}C.

Application activity 2.6

1. A shopkeeper bought 5 shirts at 8,000 Frw for each and he sold each one at 7,500 Frw.
   a) Find his loss for each shirt, write a loss as integer
   b) Calculate the total loss of 5 shirts altogether, write a loss as integer
2. A learner moved 3 steps for four times to the right side on a number line, where did she stop from if she started from 0?
3. Temperature in summer increased by +3^{o}C per hour. What was the increase in 9 hours?
4. A Football team lost 3 points in each of the last 5 games played. What was the loss altogether?

2.7. End unit assessment

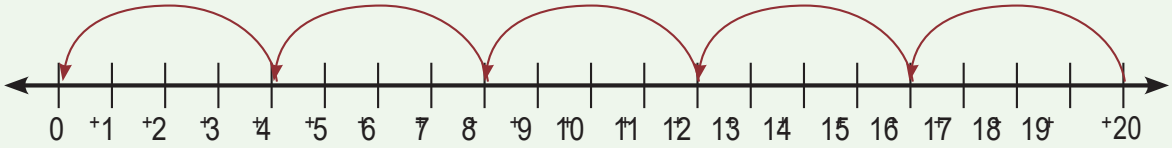
Assessment of knowledge and understanding

1. Solve without using a number line.
   (a) (+24) ÷ (+8) = (b) (-10) × (+3) = (c) (66) ÷ (- 11) =
   (d) (-12) × (-12) = (e) (12) ÷ (12)
2. State whether the following statements are true or false:
   (a) The product of two negative integers is always positive.
   (b) Dividing a positive integer by a negative integer gives a positive result.
   (c) (−5)×(−4)=+20.
3. Fill in the blanks:
   (a) (−8)×(_)= +24
   (b) (_)÷(−6)=−42
   (c) The quotient of(−45) and (+9) is ____.

Assessment of skills

4. Work out the following using a number line.
   (a) 3× (- 4) = (b) +21÷ (-3) = (c) +30÷ (+6) = (d) (+8) × (-4) =
5. Ngabire bought 4 kg of sugar each costing 1,200 Frw. How much did she pay?
6. Mukahirwa owed 12,000 Frw for each to her 4 friends.
   a) How much did she owe altogether?
   b) Write the answer as an integer.
   c) She earned 40,000 Frw and paid it off to them equally. How much is she still owed by each one?

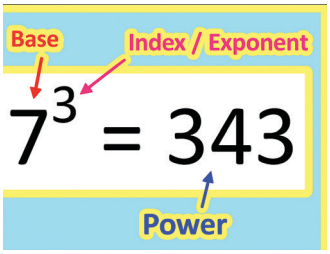
7. A debt of 1,600 Frw was cleared by 4 people. How much did each clear?
8. Uwera took a loan of 6,000 Frw. She paid it in 4 equal installments. How much was each installment?
9. A debt of 1,200 Frw was cleared equally by 6 people. How much did each person clear?
10. If a seller got profit of 30,000 Frw per day, find her profit for a week and write the answer using integers.
11. Write the division statement shown on the number line:



Assessment of attitudes and values

12. (a) Basing on the profit made by a shopkeeper per day, explain why it is important to understand the rules of multiplying and dividing integers in real life?
    (b) How would you explain the concept of "debt" using integer operations?
13. Mukahirwa owed money to her friends but could only pay a part of it.
    i) What does this situation teach about responsibility in financial matters?
    ii) How would you handle a situation where you owe money but cannot pay fully.

UNIT 3



POWERS, INDICES, LCM AND GCF

Key unit Competence: You will be able to use powers, indices and apply the Lowest Common Multiple (LCM) and the Greatest Common Factor (GCF) when solving problems.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- State and explain the law of multiplication and division of powers.
- Explain the terms "base" and "exponent" of a power.
- Identify the ways of working out problems involving the LCM in a practical context.

Skills:

- Apply the law of indices in multiplication and division of powers.
- Calculate the LCM and the GCF of numbers.
- Apply the LCM and GCF in solving problems.

Attitudes and values:

- Respect each other when working in groups on division or multiplication of powers.
- Acknowledge the importance of the law on indices when solving real life problems.
- Show the confidence and accuracy when finding the LCM and GCF of numbers.

3.0. Introduction

Mastering powers and indices along with LCM and GCF equips pupils with important mathematical concepts with numerous real-life applications such as in sports, medical subscription, and in financial situations.

Examples:

- Banks use powers to calculate interest, showing how money grows over time.
- LCM helps to identify when things happen at the same time. If one bus comes every 4 minutes and another bus every 6 minutes, the LCM tells you when both buses come together.
- GCF helps when we divide things into equal groups. If 12 apples and 16 oranges need to be shared equally, GCF tells us how many groups we can make for each type of fruits.

Introductory Activity

1. Suppose that you have 1,000 Frw. If you double your money every day for three days, you can use mathematical expressions to show that your money grows over time.

   Dou you think the following 2 mathematical expressions mean the same? Explain. $(1{,}000 \times 2 \times 2 \times 2)$ and $(1{,}000 \times 2^3)$

2. Given a price list with multiples of the cost of bananas (40 Frw) and oranges (60 Frw). If you want to buy both bananas and oranges with the same amount of money, you need to know the smallest amount you need to spend. Which technique can you use to know such amount to spend? Explain. Your answer basing on:
   - **The multiples of 40:** 40, 80, 120, 160...
   - **The multiples of 60:** 60, 120, 180, 240...

3. The distance from the earth to the sun can be easily written in short as follows: 149.6×10^6 km

   a) Have you ever come across numbers written in this form?

   b) How is such form helpful? Explain.

3.1 Meaning of indices

Activity 3.1

- Write 64 on a sheet of paper and factorize it into prime factors.
- What prime factor do you use?
- How many times did you multiply it to get 64?
- Write the prime factor once and write the number of times to its right, above it.
- Explain your working steps.

Summary

- Indices help us write repeated multiplication in a shorter way. Instead of writing $2 \times 2 \times 2$, we can simply write 2^3. The small number on top 3 tells us how many times we multiply the big number 2.
- When writing in indices:

  a) Write the number that has been multiplied repeatedly once.

  b) Count and write the number of times the number has been multiplied to the right, above that number.

Example 1

Write $3 \times 3 \times 3 \times 3$ using indices.

Solution

$3 \times 3 \times 3 \times 3 = 3^4$

(3 has been multiplied by itself 4 times. Write 4 to the right, above 3)

Example 2

Write $8 \times 8 \times 8 \times 8 \times 8$ using indices.

Solution

$8 \times 8 \times 8 \times 8 \times 8 = 8^5$

(8 has been multiplied by itself 5 times. Write 5 to the right, above 8

Application activity 3.1

1. Write the following numbers using indices:
   (a) $7 \times 7 \times 7 \times 7 \times 7 =$
   (b) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 =$
2. Complete the following statements:
   (a) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^{?}$ (b) $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 11^{?}$
3. Write the following powers in expanded form:
   (a) $3^8 =$ (b) $5^9 =$ (c) $2^4 =$

3.2 Defining base and exponent

Activity 3.2

- Study the number 7^4 and answer the questions that follow.
- Which number has been multiplied repeatedly? What is its name?
- Which number shows the number of times the number has been multiplied repeatedly? What is its name? Explain.

Summary

- The number that has been multiplied over and over is called the **base**. For example, in 2^3, the number **2** is the base because it is multiplied by itself three times ($2 \times 2 \times 2$).
- The small number above it 3 is called the **index** or **exponent**. It tells us how many times the base is multiplied.
- In general, the expression a^n of a number with a base (a) and exponent (n) is called the power.

$$\text{Base} \leftarrow a^n \rightarrow \text{Exponent}$$

Example

Given 6^3, which number shows the base and exponent and why?

Solution

6 is the base. While 3 is the exponent or index, because:

- Base is the number that has been multiplied repeatedly.
- Exponent or index is the number of times a number has been multiplied by itself

Application activity 3.2

1. State the number that represents the base in the following

   5^2 6^4 10^7

2. State the exponent in the following:

   7^5 15^9 11^5

3. Given that 9 is the base and 6 is the exponent, write the number in power notation.
4. Expand 11^7
5. Write the following in power notation: $6\times6\times6\times6\times6\times6\times6\times6\times6\times6\times6\times6\times6$
   State the exponent and the base.

3.3 The law of multiplication of indices

Activity 3.3

- Given $6^2 \text{ x } 6^5$, expand and simplify.
- How many times has 6 been multiplied?
- Now add the indices in the first expression.
- Leave your answer in power notation.
- Compare your results. What do you notice?

Summary

The law of multiplying indices states that "When multiplying numbers with the same base, maintain the base and add the exponents or indices".

Property: $a^n \times a^m = a^{n+m}$

Example 1

Simplify $7^2 \times 7^2$.

Solution

Method 1

Use expanded notation.

$7^2 \times 7^2 = (7 \times 7) \times (7 \times 7) = 7^4$

Method 2

Add the indices.

$7^2 \times 7^2 = 7^{(2+2)} = 7^4$

Example 2

Simplify $3^3 \times 3^2 \times 3^2$.

Solution

Method 1

Use expanded notation.

$3^3 \times 3^2 \times 3^2 = (3\times3\times3) \times (3\times3) \times (3\times3\times3) = 3^8$

Method 2

Add the indices.

$3^3 \times 3^2 \times 3^2 = 3^{(3+2+3)} = 3^8$

Application activity 3.3

Simplify the following: Leave your answers in power notation.

(a) $6^2 \times 6^3 =$ (b) $9^2 \times 9^2 =$ (c) $10^5 \times 10^6 =$ (d) $12^4 \times 12^5 =$ (e) $4^3 \times 4^3 \times 4^2 =$

(f) $7^6 \times 7^4 =$ (g) $20 \times 20 \times 20^5 =$ (h) $13 \times 13 =$ (i) $5^3 \times 5 =$

3.4 Law of division of indices

Activity 3.4

- Study the statement: 27 ÷ 9 and prime factorise the numbers.
- Write the numbers in power notation.
- Simplify, leaving the answer in power notation.
- Now write the first power notation expression again.
- Maintain the base and subtract the indices. Compare your answers.
- Now expand $6^5 \div 6^3 =$. Show your working steps.
- Then work out $6^{(5-3)} =$. Compare your results. What do you notice?

Summary

- When dividing powers with the same base, we can expand the powers and then divide step by step. For example: $6^4 \div 6^2 = \frac{6 \times 6 \times 6 \times 6}{6 \times 6} = 6 \times 6 = 6^2$
- When we divide numbers with the same base, we subtract their indices or exponents. For example: $6^4 \div 6^2 = 6^{(4-2)} = 6^2$.
- **Property**: $a^n \div a^m = a^{n-m}$

Example 1: Work out $4^3 \div 4^1$

Solution

Method 1: Expand and divide.

$$4^3 \div 4^1 = \frac{4 \times 4 \times \cancel{4}}{\cancel{4}} = 4^2$$

Method 2: Subtract the indices

$$4^3 \div 4^1 = 4^{(3-1)} = 4^2$$

Example 2: Work out $8^5 \div 8^3$

Solution

Method 1: Expand and divide.

$$8^5 \div 8^3 = \frac{8 \times 8 \times \cancel{8} \times \cancel{8} \times \cancel{8}}{\cancel{8} \times \cancel{8} \times \cancel{8}}$$

$$= 8^2 = 8 \times 8 = 64$$

Method 2 : Subtract the indices

$$8^5 \div 8^3 = 8^{(5-3)} = 8^2 = 8 \times 8 = 64$$

Application activity 3.4

Use both methods to simplify the following:

(a) $4^3 \div 4^1$ (b) $4^5 \div 4^3$ (c) $11^{10} \div 11^4$ (d) $3^3 \div 3^3$ (e) $6^7 \div 6^5$

3.5 Multiplying and Dividing indices

Activity 3.5

- On a slip of paper, write the numbers 32, 8 and 16 in power form.
- Multiply the powers of 32 by the powers of 8, then divide by the powers of 16.
- What answer do you get in power form?

Summary

- When multiplying and dividing indices, use both the law of multiplication and that of dividision of indices.
- When a number has zero as an exponent (0), it is equal to one

Example

Simplify: $\left(2^6 \times 2^8\right) \div 2^4$

Solution

Method 1: Expand and divide

$$\left(2^6 \times 2^8\right) \div 2^4 = \frac{2^6 \times 2^8}{2^4} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = 2^{10}$$

Method 2: Use of multiplication and division laws

$$\left(2^6 \times 2^8\right) \div 2^4 = 2^{(6+8)} \div 2^4$$

$$\frac{2^{14}}{2^4} = 2^{14-4} = 2^{10}$$

Application activity 3.5

Evaluate the following

(a) $7^5 \times 7^3 \div 7^5$ (c) $a^2 \times a^3 \div a^3$ (e) $q \times q^6 \div q^7$

(b) $6^4 \times 6 \div 6^4$ (d) $5^3 \times 5^2 \div 5^5$

3.6 Using the law of multiplying indices to find the missing exponent or index

Activity 3.6

- Given 25 and 125 two multiples of 5.
- Express them in power form.
- Multiply the two indices and write the answer in power notation.
- If $5^{?} \times 5^{3} = 5^{5}$, use the law of multiplying indices and compare indices on both sides to find the missing exponent or index.

Summary

When multiplying numbers with the same base, we add their indices. This rule helps to compare indices or exponents on both sides and find the missing exponent.

For example: Find the missing exponent in the following $4^{?} \times 4^{3} = 4^{9}$

Solution

- Apply the law of multiplying indices, we have: $4^{?} \times 4^{3} = 4^{(?+3)} = 4^{9}$
- From comparison of exponents or indices, we have the following:
- $?+3=9$ which is equivalent to $6+3=9$
- Therefore, the missing exponent or index is 6.

Application activity 3.6

Find the missing indices or exponents

(a) $7^{7} \times 7^{?} = 7^{11}$ (b) $3^{2} \times 3^{?} = 3^{3}$ (c) $5^{6} \times 5^{?} = 5^{10}$

(d) $9^{?} \times 9^{5} = 9^{9}$ (e) $6^{6} \times 6^{3} = 6^{?}$

3.7 Finding the missing number in the expression with division of indices

Activity 3.7

- Given 25 and 125 two multiples of 5.
- Express them in power form.
- Divide the two indices and write the answer in power notation.
- If $5^{?} \div 5^{3} = 5^{2}$, use the law of multiplying indices and compare indices on both sides to find the missing exponent or index.

Summary

When dividing numbers with the same base, we subtract their indices. This rule helps to compare indices or exponents on both sides and find the missing exponent.

Example 1: Find the missing exponent in the following $4^9 \div 4^? = 4^3$

Solution

- Apply the law of dividing indices, we have: $4^9 \div 4^? = 4^{(9-?)} = 4^3$.
- From comparison of exponents or indices, we have the following:
- $9 - ? = 3$ which is equivalent to $9 - 6 = 3$
- Therefore, the missing exponent or index is 6.

Example 2: Find the missing exponent in the following $5^9 \div 5^? = 5^3$

Solution

- Apply the law of dividing indices, we have: $5^9 \div 5^? = 5^{(6-?)} = 5^3$
- From comparison of exponents or indices, we have the following:
- $6 - ? = 3$ which is equivalent to $6 - 3 = 3$
- Therefore, the missing exponent or index is 3.

Application activity 3.7

Simplify the following and find the missing index or exponent.

(a) $2^3 \div 2^x = 2^2$ (b) $10^4 \div 10^p = 10^2$ (c) $6^y \div 6^1 = 6^3$

3.8 Finding the lowest common multiple (LCM) of numbers

Activity 3.8

- Write 8 and 12 on slips of paper.
- Write down the first ten multiples of each number.
- Identify their common multiples.
- What is their lowest common multiple?

Summary

When finding the Lowest Common Multiple (LCM) of numbers, there are 2 methods:

- **Method 1:** List the multiples of the given numbers and choose the lowest common multiple.
- **Method 2**: Prime factorise the given numbers then find the product of all the prime factors.

Example: Find the LCM of 6 and 9

Solution

Method 1: List multiples of 6 and 9

- Multiples of 6 are: 6, 12, 18, 24,30,36,42, ...
- Multiples of 9 are: 9, 18, 27, 36, 45, 54, ...
- The common multiples are 18, 36, ...
- The LCM of 6 and 9 is 18

Method 2: Prime factorise 6 and 9

	6
2	3
3	1

$6 = 2 \times 3$

	9
3	3
3	1

$9 = 3 \times 3$

LCM= $2 \times 3 \times 3 = 18$

Application activity 3.8

Find the LCM of the following:

(a) 8 and 6 (b) 18 and 16 (c) 7 and 11

(d) 25, 30 and 45 (e) 45, 60 and 70

3.9 Solving problems involving LCM

Activity 3.9

- Two alarm clocks ring in intervals of 10 and 15 minutes.
- Make a list of time intervals for each of the two clocks.
- What is the first common time interval of the two clocks?
- What does the first common time interval mean?

Summary

To solve problems involving the lowest common Multiples (LCM) of numbers:

- List the multiples of the given numbers then choose the lowest common multiple.
- Read, understand and solve the question thoroughly.

Example

Mugisha was diagnosed a malaria disease and the doctor prescribed the treatment as follows: 4 tablets of Coartem to be swallowed every 12 hours and 2 tablets of Cotrimazole to be swallowed every 8 hours. How long will Mugisha swallow both drugs together again?

Solution

- Multiples of 12 are 12, 24, 36, 48, 60, 72, ...
- Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, ...
- Common multiples are 24 and 48.
- Lowest Common Multiple (LCM) is 24.
- Therefore, Mugisha will swallow both medicines together again after 24 hours.

Application activity 3.9

1. Three taxis leave the park at intervals of 15, 20 and 25 minutes. After how long will the taxis leave the park at the same time?
2. Ganza packed books in boxes that carry 15 books each. Ines could not carry the boxes because they were too heavy for her. She repacked the books in boxes carrying 9 books each. How many books were there if she did not leave any book out?
3. Two nurses administer medication to two patients in intervals of 30 and 45 minutes respectively. How long will the nurses administer medication to the patients at the same time if they started at the same time?
4. Three buses arrive at the bus park at intervals of 30, 40 and 45 minutes. How long will the buses take to arrive at the park at the same time if their first arrival time was the same?


Buses arriving at the bus park

3.10 Finding the greatest common factor (GCF) of numbers

Activity 3.10

- List three 2 digit-numbers of your choice.
- Find the factors of each number.
- List the factors that are common.
- What is the greatest of all the common factors?

Summary

To find the greatest common factor (GCF) of numbers:

- List all the factors of the given numbers.
- Pick out the common factors then the greatest common factor.
- You can also use a factor tree to find the common factors, then work out their product.
- Consider the prime common factors for all members but with smaller exponent.

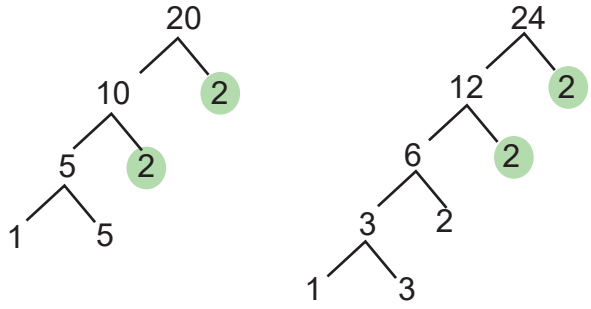
Example: What is the greatest common factor (GCF) of 20 and 24?

Solution

Method 1: List the factors of 20 and 24.

- Factors of 20 are 1, 2, 4, 5, 10, and 20.
- Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.
- The common factors are 1, 2 and 4. The GCF is 4

Method 2: Prime factorization of 20 and 24 to get the common prime factors.



- Take the common factor with small exponent. It is $2^2 = 4$.
- GCF is 2 x 2 = 4.
- Therefore, the GCF of 20 and 24 is 4

Application activity 3.10

1. Find the greatest common factor (GCF) of the following numbers using lists of factors.
   (a) 30 and 40 (b) 120 and 180 (c) 60, 120 and 180 (d) 45, 60 and 75
2. Find the GCF of the following by using prime factorization
   (a) 64 and 128 (b)120, 220 and 360

3.11 Solving problems involving GCF

Activity 3.11

- Two wires 448 cm and 616 cm in length are to be cut into pieces of the same length without any remainder.
- What do we do to find the greatest possible length of the pieces?
- Try to work out the problem. Explain your working steps.

Summary

1. To solve problems involving the lowest common Multiples (LCM) of numbers. The Greatest Common Factor (GCF) is the biggest number that divides evenly into two or more numbers. It helps us solve problems, especially when we need to divide things into equal groups or simplify fractions to the lowest term. For example, GCF is useful in packaging products in factories.
2. To solve problems involving GCF, we follow these steps
   - List the factors of each number.

- Find the common factors for both numbers
- Pick the greatest one which is the GCF for both numbers.

Example

Emmanuel has 12 red balloons and 18 blue balloons. She wants to group them equally. How many balloons should be in each group?

Solution

1. List the factors:
   - Factors of 12: 1, 2, 3, 4, 6, 12
   - Factors of 18: 1, 2, 3, 6, 9, 18
2. Find the common factors: 1, 2, 3, 6
3. The greatest common factor is 6, so the GCF is 6.
4. Emmanuel can make 2 and 3 groups of 6 balloons, with each group having both red and blue balloons.

Application activity 3.11

1. What is the greatest number that can divide 36 and 54 without leaving a remainder?
2. Mutesi collects 30 litres of milk from her farm in the morning. She collects 35 litres of milk in the afternoon and 45 litres in the evening. What is the capacity of the biggest container that can be used in all instances with no milk remaining?
3. Musa collected 48 kg of okra seeds from one garden and 84 kg from another. Find the mass of the pack that can be used in both instances without leaving any okra seeds in the garden
4. The workers at a maize mill have 3 sacks of maize flour weighing 90 kg, 120 kg and 150 kg respectively. What is the mass of the biggest packs that can be used so that no flour remains in any of the sacks?

3.12 Finding the missing number using LCM and GCF

Activity 3.12

- Write two numbers of your choice on slips of paper.
- Work out their LCM.
- Now find the GCF of the two numbers you chose.
- Calculate the product of the two numbers, then divide by the LCM. What do you observe?
- Now calculate by dividing the previous product by the GCF.
- What is the relationship between the product, GCF and LCM?

Summary

To find a missing number when given the LCM (Least Common Multiple), GCF (Greatest Common Factor), and another value, follow these steps:

1. Identify what is given: you have an LCM, a GCF, and one known number.
2. Use the relationship: the product of the LCM and GCF of two numbers equals the product of those two numbers.
3. Find the missing number using division.

Example1

The LCM of two numbers is 30. One of the numbers is 10. If the GCF is 5, find the second number.

Solution

GCF = 5, LCM =30, Given number = 10, Missing number =?

$$\text{Missing Number} \times \text{Given Number} = \text{LCM} \times \text{GCF}$$

$$\text{Missing Number} = \frac{\text{LCM} \times \text{GCF}}{\text{Given Number}}$$

$$\text{Missing Number} = \frac{30 \times 5}{10} = \frac{150}{10} = 15$$

Therefore, the second number is 15

Example 2

Lisa and Tom are organizing a sports event. They need to buy whistles for referees. Lisa buys a set of whistles with a total LCM of 48, and Tom buys another set where the GCF is 4. One of their sets has a total of 16 whistles. How many whistles are in the other set?

Solution

GCF = 4, LCM =48, Given number = 16, Missing number =?

$$\text{Missing Number} \times \text{Given Number} = \text{LCM} \times \text{GCF}$$

$$\text{Missing Number} = \frac{\text{LCM} \times \text{GCF}}{\text{Given Number}}$$

$$\text{Missing Number} = \frac{48 \times 4}{16} = \frac{192}{16} = 12$$

Therefore, the other set contains 12 whistles.

Application activity 3.12

1. The LCM of two numbers is 72. Find the other number if one of the numbers is 8 and GCF is 12
2. The GCF of two numbers is 14. Find the other number if one of the numbers is 42 and LCM is 84
3. The LCM of two numbers is 144. The GCF is 2. If one of the numbers is 18, what is the other number?
4. The GCF of two numbers is 3. The LCM is 60. If one of the numbers is 12, find the second number.

3.13. End unit assessment

Assessing knowledge and understanding

1. Work out the following:

   (a) $8^4 \div 8^3 =$ _____ (b) $3^3 \times$ _____ $= 3^5$ (c) ___ $\times 7^2 = 7^3$

   (d) $5^5 \div$ _____ $= 5^2$ (e) $12^3 \times 12^2 =$ ___ (f) $13^4 \times$ ___ $\div 13^2 = 13^5$

2. Define the following terms:

   (a) LCM (Least Common Multiple) of numbers.

   (b) GCF (Greatest Common Factor) of numbers.

3. True or False?

   (a) The LCM of two numbers is always greater than or equal to both numbers.

   (b) The GCF of two numbers cannot be larger than the smaller number.

   (c) $a^m \times a^n = a^{m+n}$ for any non-zero base a and any real numbers n and m.

Assessment of skills:

4. Find the LCM and GCF of the following:

   (a) 60 and 120 (b) 36, 48 and 92 (c) 9, 15 and 36

5. What is the GCF and LCM of the two numbers 9 and 15?

6. John and Jesca were each given sugarcane of equal length. John cut his sugarcane into equal lengths of 20 cm, while Jesca cut her sugar cane into equal lengths of 50 cm. If they don't have any remainder, find the shortest possible length of sugarcane should each equally have

7. At a factory, 3 bells ring at intervals of 15 minutes, 20 minutes and 30 minutes to mark shifts of different departments. If they first ring together at 8:00 am. When do they ring together again?

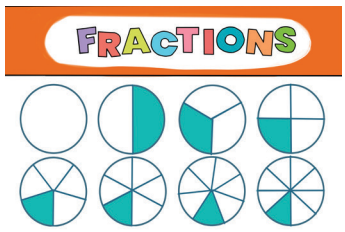
8. Uwera prepared 96 litres of orange juice, 72 litres pineapple juice and 84 litres of watermelon juice she kept it small containers. What is the capacity of the biggest container she used if no juice remained?

9. At exactly 6:00 a.m, two buses of 2 different companies leave the bus terminal. Thereafter, every 40 and 70 minutes, a bus leaves the terminal. Find the time the buses will leave the terminal together.

Assessing attitudes and values

10. At Rwamagana car station, a bus of STELLA leaves the station every 30 minutes and a bus of International leaves the station after every 45 minutes. The coordinator of this car station uses numbers to follow the movement of buses and clients. Is it necessary to study the LCM of numbers? How does calculating LCM improve public transport scheduling?

UNIT 4



OPERATIONS ON FRACTIONS

Key unit Competence: You will be able to apply fractions in daily life situations to solve related problems.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain how to multiply and divide fractions.
- Describe how to solve problems involving fractions

Skills:

- Multiply and divide fractions.
- Solve word problems involving fractions.

Attitudes and values:

- Appreciate the importance of fractions in daily life situations.
- Show confidence and accuracy when solving problems involving fractions.

4.0. Introduction

Applying fractions in a daily life is not only enhancing mathematical understanding but also equips you with essential skills needed for everyday tasks such as cooking, budgeting, managing time effectively, and analysing sports statistics. By solving real-life problems involving fractions, you can develop critical thinking skills that will serve you well beyond the classroom.

Introductory Activity

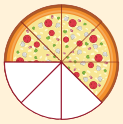
Suppose that a pineapple is shared equally among 3 people.

a) What portion/fraction does each person get?
b) If two of them decide to combine their portions, what mathematical operations do they carry out?
c) Find out some examples of equal sharing involving Mathematical operations in real life, and present them using fractions.

4.1 Multiplying a whole number by a fraction

Activity 4.1

- Imagine you have a bread with 8 slices.



a) If you want to give $\frac{1}{8}$ of the bread to every child, how many slices will you give to 5 children?

b) What fraction of bread will be given to all of them?

Summary

To multiply a whole number by a fraction:

- Multiply the whole number by the numerator, then divide the product by the denominator.
- Simplify where necessary or write a whole number as a fraction by putting 1 as its denominator then multiply.

Example

Multiply: $15 \times \frac{2}{3}$

Solution

Write 1 under 15 as its denominator

$$15 \times \frac{2}{3} = \frac{15}{1} \times \frac{2}{3} = \frac{30}{3} = 10$$

Application activity 4.1

a) Multiply 169 by $\frac{4}{13}$

b) What is the product of 62 by $3\frac{1}{4}$?

c) Calculate the product of 102 by $4\frac{1}{5}$.

4.2 Multiplying a fraction by a whole number

Activity 4.2

Look at the sugarcane below. It is a whole divided in equal parts.

a) How many parts does the sugarcane have?

b) Write the fraction representing one part.
c) Multiply the fraction above in (b) by the number of parts.
d) What do you obtain?

Summary

Example1

Multiply: $\frac{5}{12} \times 24$

Solution

Write 1 under 24 as its denominator

$$\frac{5}{12} \times 24 = \frac{5}{12} \times \frac{24}{1} = \frac{5 \times 24}{12} = \frac{120}{12} = 10$$

Example 2: $\frac{3}{4}$ of 20 is $\frac{3}{4}(20) = \frac{3 \times 20}{4} = \frac{60}{4} = 15$

Application activity 4.2

1) What is $\frac{1}{4}$ of 20 mangoes?

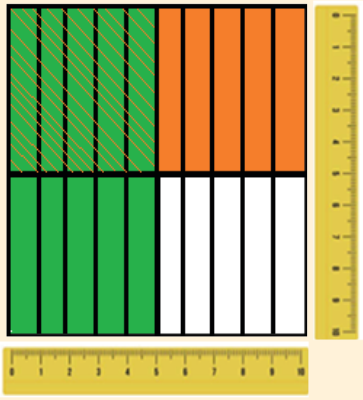
2) Multiply: $\frac{3}{7} \times 7$

3) What is the product of $1\frac{2}{5}$ and 15 ?

4.3. Multiplying a fraction by a fraction

Activity 4.3

- Draw a square of 10 cm of side. It represents a whole.
- Divide it into 10 equal parts using vertical lines.
- Shade 5 parts of them.
- Now divide each part into halves using an horizontal line.
- Shade $\frac{1}{2}$ of the square.
- Count and form a fraction of the double shaded part out of all parts of the whole. What is your answer?
- Now multiply $\frac{1}{2}$ by $\frac{5}{10}$. What do you get?
- Compare the product and the fraction of the double shaded part.



Summary

When multiplying a fraction by a fraction, multiply both numerators separately and then the denominators separately. Simplify where possible.

To multiply a fraction by a whole number:

- Multiply the numerator by the whole number,
- Copy the denominator,
- Simplify the obtained fraction.

$$\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$$

Example

Simplify $\frac{5}{6}\times\frac{3}{4}$

Solution

$$\frac{5}{6}\times\frac{3}{4}=\frac{5\times3}{6\times4}=\frac{15}{24}=\frac{5}{8}$$

Application activity 4.3

(1) What is $2\frac{1}{4}$ multiplied by $1\frac{1}{2}$?

(2) Simplify: $\frac{1}{4}\times\frac{2}{3}\times\frac{1}{9}$

(3) Evaluate: $\frac{1}{10}\times\frac{9}{4}\times3\frac{6}{17}$

4.4 Finding reciprocals

Activity 4.4

- Write a number and multiply it by any symblol to represent a missing number.
- Equate it to one (1) to find the missing number.
- Calculate the value of the missing number.
- What do you observe about the number you wrote at first compared to the result?

5 x ? = 1

Summary

The reciprocal of a given number is the other number that is multiplied by it to get 1 as a product.

Examples

1. What is the reciprocal of 5?

Solution

- Knowing that a number multiplied by its resprocal gives 1, we write the following equality: 5 ×? = 1
- The missing number or reciprocal number is obtained by dividing 1 by 5.
- The reciprocal of 5 is $\frac{1}{5}$ because $5\times\frac{1}{5}=\frac{5\times1}{5}=\frac{5}{5}=1$

2. Find the reciprocal of $\frac{1}{3}$

Solution

- Knowing that a fraction multiply by its resprocal give 1, we write the following equality: $\frac{1}{3}$ × ? = 1
- The missing number is obtained by multiplying 1 by the denominator of the fraction and then divide by the numerator.

$$\text{The resprocal number}=\frac{1\times 3}{1}=3$$

- The reciprocal of $\frac{1}{3}$ is 3.

Note: The reciprocal of a fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$ if a, b are not zero.

Application activity 4.4

Find the reciprocal of the following:

(a) 4 (b) 25 (c) $\frac{3}{7}$ (d) $\frac{8}{11}$ (e) $2\frac{3}{4}$

Examples: Reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ because $\frac{3}{4}$ x $\frac{4}{3}$ = $\frac{3\text{ x }4}{4\text{ x }3}$ = 1

4.5 Dividing a whole number by a fraction

Activity 4.5

Imagine you have 4 chocolate bars, and you want to share all of them by giving half a bar to each friend.

- How many friends can get a piece that is equal to half a bar?
- How do you work out the following: $4\div\frac{1}{2}=?$. Explain your working steps.

Summary

To divide a whole number by a fraction:

- Multiply the whole number by the reciprocal of the fraction.
- Simplify where necessary.

Example 1

Divide 32 by $\frac{3}{5}$

Solution

Multiply the whole by the reciprocal of $\frac{3}{5}$

$$32 \div \frac{3}{5} = 32 \times \frac{5}{3} = \frac{32}{1} \times \frac{5}{3} = \frac{160}{3} = 53\frac{1}{3}$$

Example 2

Divide 38 by $1\frac{8}{11}$

Solution

Express the mixed fraction as an improper fraction

$$38 \div 1\frac{8}{11} = 38 \div \frac{19}{11}$$

$$= 38 \times \frac{11}{19} = \frac{\overset{2}{\cancel{38}}}{1} \times \frac{11}{\cancel{19}_{1}} = 2 \times 11 = 22$$

Application activity 4.5

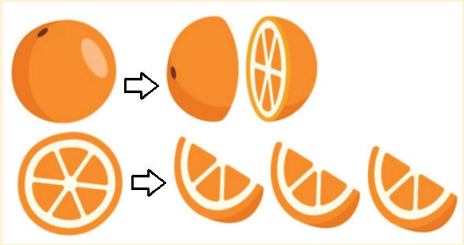
Workout the following:

(a) $4 \div \frac{1}{2}$ (b) $10 \div 3\frac{1}{3}$ (c) $25 \div \frac{5}{11}$ (d) $60 \div 2\frac{1}{2}$ (e) $96 \div 2\frac{2}{7}$

4.6 Dividing a fraction by a whole number

Activity 4.6

- Draw an orange and divide it into two equal parts. What fraction do you have?
- Divide each portion into 3 portions. What is the fraction of one portion out of all the portions of a whole orange?
- What happens if you share equally a half of an orange to 3 friends? Does each of your friends get a big part than a half? What do you notice?



Summary

To divide a fraction by a whole number, first write the fraction, then multiply the fraction by the reciprocal of the whole number.

Example 1

Divide $\frac{1}{2}$ by 3

Solution

the reciprocal of 3 is $\frac{1}{3}$

$$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Example 2

workout: $3\frac{1}{4} \div 13$

Solution

Express the mixed fraction as an improper fraction:

$$3\frac{1}{4} \div 13 = \frac{13}{4} \div 13 = \frac{\cancel{13}}{4} \times \frac{1}{\cancel{13}} = \frac{1}{4}$$

Application activity 4.6

Workout the following:

a) $\frac{5}{6} \div 7$ b) $\frac{3}{4} \div 9$ c) $2\frac{4}{5} \div 4$ d) $2\frac{1}{2} \div 4$ e) $1\frac{1}{4} \div 5$

4.7 Dividing a fraction by a fraction

Activity 4.7

- Draw a circle and divide it into 8 equal portions.
- Shade 3 portions and write down the fraction of the shaded part.
- Divide the shaded portion into 2 equal parts.
- Shade one portion. Write it as a fraction of the first shaded portion.
- Divide the circle into more equal small portions.
- How many are there?
- What is the relationship between the first and final shaded parts?
- What is the fraction of the final shaded portion out of all the small portions of the circle? Write it down.

Summary

When dividing a fraction by a fraction:

- Multiply the first fraction by the reciprocal of the second fraction.
- Simplify where possible.

Example 1

workout $\frac{3}{4} \div \frac{1}{3}$

Solution

reciprocal of $\frac{1}{3}$ is $\frac{3}{1}$ then $\frac{3}{4} \div \frac{1}{3} = \frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$

Example 2

Divide: $4\frac{2}{3} \div 1\frac{3}{4}$

Solution

Express the mixed fractions as an improper fraction

$4\frac{2}{3} \div 1\frac{3}{4} = \frac{14}{3} \div \frac{7}{4}$

the reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$

$\frac{14}{3} \div \frac{7}{4} = \frac{14}{3} \times \frac{4}{7} = \frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$

Application activity 4.7

Workout the following

(a) $\frac{3}{4} \div \frac{2}{9}$ (b) $1\frac{1}{3} \div \frac{4}{7}$ (c) $\frac{1}{4} \div 2\frac{3}{8}$ (d) $5\frac{1}{3} \div 2\frac{1}{4}$ (e) $\frac{56}{57} \div \frac{19}{44}$

4.8 Multiplying and dividing fractions

Activity 4.8

- Write three fractions of your choice.
- Multiply the first two.
- Divide the product of the first two by the third fraction.
- Which steps did you carry out to get the answer? Explain.

Summary

- When calculating combined multiplying and dividing fractions, work out division first, then multiply and simplify where possible. Use BODMAS.
- Multiplication and division of fractions can be worked by changing the divided fraction to its reciprocal. Then the fractions are multiplied together.

Example 1

Workout $\frac{1}{2}\times\frac{2}{3}\div\frac{5}{6}$

Solution

Method 1

Workout division first. (BODMAS)

$$\frac{2}{3}\div\frac{5}{6}=\frac{2}{3}\times\frac{6}{5}=\frac{12}{15}$$

Then multiply

$$\frac{1}{2}\times\frac{12}{15}=\frac{12}{30}=\frac{2}{5}$$

Method 2

Combine them and remember to change a divisor into reciprocal

$$\frac{1}{2}\times\frac{2}{3}\div\frac{5}{6}=\frac{1}{2}\times\frac{2}{3}\times\frac{6}{5}=\frac{12}{30}=\frac{2}{5}$$

Example 2

Workout: $1\frac{1}{3}\div\frac{3}{4}\times2\frac{1}{2}$

Solution

Method 1

Workout division first. (BODMAS)

And remember to express the mixed fractions as an improper fraction

$$1\frac{1}{3}\div\frac{3}{4}=\frac{4}{3}\div\frac{3}{4}=\frac{4}{3}\times\frac{4}{3}=\frac{16}{9}$$

Then multiply

$$\frac{16}{9}\times2\frac{1}{2}=\frac{16}{9}\times\frac{5}{2}=\frac{80}{18}=\frac{40}{9}=4\frac{4}{9}$$

Method 2

Combine them and remember to change a divisor into reciprocal

$$1\frac{1}{3}\div\frac{3}{4}\times2\frac{1}{2}=1\frac{1}{3}\times\frac{4}{3}\times2\frac{1}{2}=\frac{4}{3}\times\frac{4}{3}\times\frac{5}{2}=\frac{80}{18}=\frac{40}{9}=4\frac{4}{9}$$

Application activity 4.8

Workout the following

(a) $6\frac{1}{3}\div3\frac{2}{3}\times2\frac{1}{4}$ (b) $\frac{9}{10}\div1\frac{1}{4}\times2\frac{1}{3}$ (c) $3\frac{1}{2}\times\frac{4}{5}\div1\frac{1}{3}$ (d) $\frac{1}{3}\times3\frac{2}{3}\div\frac{1}{4}$ (e) $\frac{3}{4}\times\frac{1}{2}\div\frac{1}{3}$

4.9 Solve problems involving multiplication and division of fractions

Activity 4.9

Mbabazi and Rugari stood for the post of Chairperson for their saving group. Mbabazi got $\frac{1}{3}$ of the votes and Rugari got the rest. If 3,600 people voted:

a) How many people voted for Mbabazi?

b) How many people voted Rugari? What fraction presents Rugari's votes?

Summary

- In order to work out problems involving multiplying and dividing fractions, read and interpret in order to understand.
- Identify the operations to be used.

Example 1

Uwera had a land. She gave $\frac{2}{5}$ of it to her son, Kwizera. She also gave $\frac{3}{8}$ of the land to her daughter, Cissy.

a) What fraction of the land did she remain with?

b) If Uwera remained with 18 hectares, calculate the hectares she had originally.

Solution

Kwizera got $\frac{2}{5}$,Cissy got $\frac{3}{8}$

Land is represented by 1

$$\text{Kwizera got} = 1 - \frac{2}{5} = \frac{1}{1} - \frac{2}{5} = \frac{5-2}{5} = \frac{3}{5}$$

After giving kwizera, she remained with $\frac{3}{5}$

$$\text{Cissy got} = \frac{3}{5} - \frac{3}{8} = \frac{24-15}{40} = \frac{9}{40}$$

After giving Cissy, she remained with $\frac{9}{40}$

She remained with $\frac{9}{40}$

Uwera remained with $\frac{9}{40}$ as a fraction and 18 ha as a quantity

$$\frac{9}{40}\text{of whole land} = 18\text{ha} \leftrightarrow \text{Whole land} = \frac{40 \times 18ha}{9} = 80ha$$

Therefore, Uwera originally had 80ha.

Example 2

Moses spent $\frac{1}{4}$ of his money on food. He also spent $\frac{1}{3}$ of the remaining money on transport. He was left with 12,000 Frw. How much money did he have originally?

Solution

On food $\frac{1}{4}$, On transport= $\frac{1}{3}$ of the remainder

His money is represented by 1

$$\text{Remaining money after spend on food} = 1 - \frac{1}{4} = \frac{1}{1} - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

After spending on food, he remained with $\frac{3}{4}$

$\text{Transport} = \frac{1}{3} \text{of} \frac{3}{4} = \frac{1}{4}$

$\text{Remaining money after spend on transport} = \frac{3}{4} - \frac{1}{4} == \frac{1}{2}$

After spending on food, he remained with $\frac{1}{2}$

Moses remained with $\frac{1}{2}$ as a fraction and 12,000 Frw ha as a quantity

$\frac{1}{2} \text{of the total money} = 12{,}000\,\text{Frw} \leftrightarrow \text{The total money} = 2 \times 12{,}000\,Frw = 24{,}000\,Frw$

Therefore, Moses originally had 24,000 Frw.

Application activity 4.9

(1) The product of two number is $\frac{7}{3}$. The fist number is $\frac{7}{9}$. Find the second number.

(2) A learner attempted 30 questions in an examination. This was $\frac{3}{5}$ whole examination. How many questions were in whole examination paper?

(3) A teacher used 75 pieces of chalk to write on the blackboard for a month. This is $\frac{3}{4}$ of the full box of chalk. How many pieces of chalk does the box contain?

4.10. End unit assessment

Assessing knowledge and understanding:

1. Work out the following

   (a) $\frac{2}{3} \times \frac{2}{5}$ (b) $1\frac{3}{5} \div \frac{1}{3}$ (c) $\frac{1}{3} \times 3$ (d) $2\frac{2}{5} \div 4$ (e) $2\frac{2}{3} \times \frac{2}{5} \div 1\frac{1}{4}$.

2. Define what a reciprocal of a fraction is and give an example.
3. What is the difference between a proper and an improper fraction?
4. What is the inverse of the fraction $\frac{2}{5}$?

Assessing skills:

5. Mutesi's land is $2\frac{1}{2}$ times of Uwacu's land. Uwacu has 5 hectares. How big is Mutesi's land?
6. Akida's salary is 640,000 Frw. He spends $\frac{3}{5}$ on food, $\frac{1}{4}$ of it on school fees and $\frac{3}{20}$ on other requirements. How much does he spend on each item?
7. Reverend Kalisa was traveling on a journey. After covering $\frac{4}{9}$ of it, his car broke down. He continued the remaining 20 km by bus. Find the distance of the whole journey.

8. Sylvia made a mug of tea. $\frac{3}{5}$ of it was milk. The remaining 200 ml were water. How many millilitres filled the cup?

9. A rectangular field is $\frac{7}{8}$km long and $\frac{3}{4}$km wide. Calculate its area.

10. If $\frac{2}{3}$ of the school fees is 24,000Frw, what is the school fees?

Assessing attitudes and values:

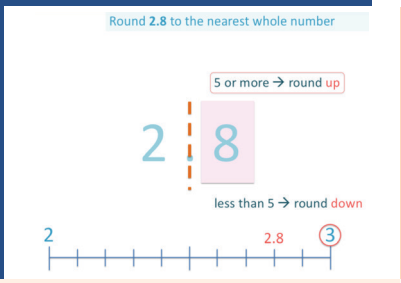
11. Read and write a short answer:

    a) Akida spends part of her salary on school fees. Why is it important to prioritize education when planning how to spend money?

    b) Two men shared the rice but one of them got a smaller fraction. How would you ensure fairness when sharing resources among people?

    c) Reverend Kalisa continued his journey by bus after his car broke down. What does this teach us about overcoming challenges?

UNIT 5



ROUNDING AND CONVERSION OF DECIMAL NUMBERS

Key Unit Competence: You will be able to round off decimals accurately, convert between fractions and decimals, and demonstrate understanding by matching equivalent fractions and decimals.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain how to round off decimal fractions.
- Describe the various steps taken when rounding off numbers.
- Illustrate and explain how to match fractions and decimals.

Skills:

- Convert fractions to decimals and vice versa.
- Explain how to round off numbers.
- Apply the knowledge acquired to match fractions and decimals.
- Carry out various calculations on rounding numbers.

Attitudes and values:

- Show confidence and accuracy when rounding of decimal numbers.
- Appreciate the use of rounding off decimal numbers in some situations and not in other situations.

Introductory Activity

Let's consider an example of school party where Primary 6 learners contributed to buy all items needed. After planning all needed items, they found that the quantity of rice needed for the party is 40.97 kg which is near 50 kg. Then they decided to buy 50 kg of rice instead of 49.97 kg.

a) Why do you think Primary 6 learners prefer to buy 50 kg instead of 49.97 kg?

b) Do you think both quantities: 50 kg and 49.97 kg are easily memorized?

c) How is rounding off useful in daily life?

5.1 Rounding off decimal numbers to the nearest tenths

Activity 5.1

Pick a flash card from read the number, identify the tenths and then match it with its corresponding card in B containing a value nearest to it.

A	B
0.243	10.1
66.98	27.2
199.605	0.2
27.166	67.0
10.09	0.6
0.5623	199.6

What have you noticed?

Summary

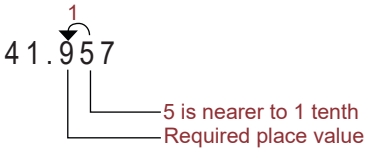
- To round off decimal to the nearest tenths, first identify the place value of tenths, then find the digit to its right.
- If the digit to the right is 0, 1, 2, 3 or 4, add 0 to the digit in the tenths place value. This is because the digits are nearest to 0.
- If the digit to the right is 5, 6, 7, 8 or 9, add 1 to the digit in the tenths place value. This is because the digits are nearest to 10.

The rounded off answer must be to one decimal place.

Example 1

Round off 41.957 to the nearest tenths.

Solution



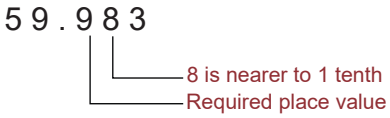
$$41.957 = \begin{array}{r} 41.95^{1}\cancel{7} \\ +\ 1 \\ \hline 42.0 \end{array}$$

Therefore, 41.957 is rounded off to 42.0.

Example 2

Round off 59.983 to the nearest tenths.

Solution



$$59.98 = \begin{array}{r} 59.9^{1}\cancel{8} \\ +\ 1 \\ \hline 60.0 \end{array}$$ *(Because 8 is nearer to 1*

Therefore, 59.983 is rounded off to 60.0

Application activity 5.1

1. Round off to the nearest tenths
   (a) 0.86 (b) 8.51 (c) 12.97
2. Round off the following to the nearest underlined place value
   (a) 56,748.93 (b) 875.39 (c) 264,537.299

5.2. Rounding off decimal numbers to the nearest hundredths

Activity 5.2

Read, think and choose the correct answer, then explain.

123.1234 rounded to the nearest hundredths is equal to. a) 123.12 b) 123.13

239.5589 rounded to the nearest hundredths is equal to. a) 239.55 b) 239.56

Explain the difference between those two numbers and how to round off a decimal number to the nearest hundredths.

Summary

- To round off a decimal to the nearest hundredths, first identify the place value of hundredths. Then find the digit to its right.
- If the digit to the right is 0, 1, 2, 3 or 4, add 0 to the digit in the hundredths place value. This is because the digits are nearest to 0.
- If the digit to the right is 5, 6, 7, 8 or 9, add 1 to the digit in the hundredths place value.

The rounded off answer must be to two decimal places.

Example 1

Round off 0.107 to the nearest hundredths.

Solution

0 . 1 0 7
7 is nearer to 1 hundredth
Required place value

$$0.107 = \begin{array}{r} 0.107 \\ +\ 1 \\ \hline 0.11 \end{array}$$

Therefore, 0.107 is rounded off to 0.11

Example 2

Round off 123.123 to the nearest hundredths.

Solution

1 2 3 . 1 2 3
3 is nearer to 0 hundredth
Required place value

$$123.123 = \begin{array}{r} 123.123 \\ +\ 0 \\ \hline 123.12 \end{array}$$

Therefore, 123.123 is rounded off to 123.12.

Application activity 5.2

1. Round off the following decimals to the nearest hundredths

   (a) 16.597 (b) 4.221 (c) 0.571

5.3. Rounding off decimal numbers to the nearest thousandths

Activity 5.3

Pick a flash card from A. Read its number, identify the thousandths and a digit right to it. Then match it with the corresponding card in B containing a value nearest to that number from A.

A	B
10.2435	10.244
66.9804	27.167
199.6057	0.2
27.1665	66.980
10.0928	0.562
0.5623	199.606

What have you noticed?

Summary

To round off a decimal to the nearest thousandths, identify the place value of thousandths and the digit to its right. Then round off considering that the digit is less or greater than 5

Example 1

Round off 5.6877 to the nearest thousandths.

Solution

5 . 6 8 7 7

7 is nearer to 1 thousandth

Required place value

$$5.6877 = \begin{array}{r} 5.6877 \\ +\quad 1 \\ \hline 5.688 \end{array}$$

Therefore, 5 . 6 8 7 7 is rounded off to 5.688

Example 2

Round off 142.1353 to the nearest thousandths.

Solution

1 4 2 . 1 3 5 3 4

3 is nearer to 0 thousandth

Required place value

$$142.13534 = \begin{array}{r} 142.13534 \\ +\quad 0 \\ \hline 142.135 \end{array}$$

Therefore, 142.13534 is rounded off to 142.135

Application activity 5.3

1. Round off the following decimals to the nearest thousandths

   (a) 2.7985 (b) 4.12.3421 (c) 5.6874

5.4. Rounding off decimal numbers to the nearest ten thousandths

Activity 5.4

Study the number cards below:

7.04683 0.62437 1,625.00016 0.99685

Write the approximate values of the numbers estimated to ten thousandths. (with 4 decimal places.

Summary

To round off decimal to the nearest ten thousandths, first identify the place value of ten thousandths. Then find the digit to its right.

If the digit to the right is 0, 1, 2, 3 or 4, add 0 to the digit in the ten thousandths place value. This is because the digits are nearest to 0.

If the digit to the right is 5, 6, 7, 8 or 9, add 1 to the digit in the ten thousandths place value.

The rounded off answer must be to four decimal places.

Example 1

Round off 3.34507 to the nearest ten thousandths

Solution

3 . 3 4 5 0 7

7 is nearer to 1 ten thousandths

Required place value

3.34507 = 3 . 3 4 5 0 7 + 1 = 3 . 3 4 5 1

Therefore,3.34507 is rounded off to 3.3451

Example 2

Round off 0.07124 to the nearest thousandths.

Solution

0 . 0 7 1 2 4

4 is nearer to 0 ten thousandths

Required place value

0.07124 = 0 . 0 7 1 2 4 + 0 = 0 . 0 7 1 2

Therefore, 0.07124 is rounded off to 0.0712

Application activity 5.4

Round off the following decimals to the nearest ten thousandths

(a) 0.06951 (b) 12.106798 (c) 482.00311 (d) 0.002456 (e) 4.99999.

5.5 Rounding off decimal numbers to the nearest hundred thousandths

Activity 5.5

Study the decimal numbers in A and B.

A	B
0.063768	0.99365
4.739256	18.88235
13.004021	4.73926
0.993652	0.06377
18.882346	13.00402

Match the corresponding decimal numbers in A to those in B.

Defend your answers.

Summary

- To round off decimal to the nearest hundred thousandths, first identify the place value of hundred thousandths. Then find the digit to its right.
- If the digit to the right is 0, 1, 2, 3 or 4, add 0 to the digit in the hundred thousandths place value. This is because the digits are nearest to 0.
- If digit to the right is 5, 6, 7, 8 or 9, add 1 to the digit in the hundred thousandths place value.

The rounded off answer must be to five decimal places.

Example 1

Round off 0.076423 to the nearest hundred thousandths

Solution

0 . 0 7 6 4 2 3

3 is nearer to 0 hundred thousandth

Required place value

0.076423 =

	0 . 0 7 6 4 2 3
+	0
	0 . 0 7 6 4 2

0.076423 is rounded off to 0.07642

Example 2

What is 76.2569293 rounded off to the nearest hundred thousandths?

Solution

7 6 . 2 5 6 9 2 9 3

9 is nearer to 1 hundred thousandth

Required place value

76.2569293 =

	7 6 . 2 5 6 9 2 9 3
+	1
	7 6 . 2 5 6 9 3

76.2569293 is rounded off to 76.25 693.

Application activity 5.5

Round off the following to the nearest hundred thousandths:

a) 0.1010110 b) 0.063285 c) 0.006192 d) 15.73912 e) 92.093018

5.6. Rounding off decimal numbers to the nearest millionths

Activity 5.6

Pick five cards with 7 and 8 decimal places from the pack.

Study each card critically.

On slips of paper, write the approximate of each number to six decimal places.

What did you consider when approximating?

Summary

To round off decimal to the nearest millionths, first identify the place value of millionths. Then find the digit to its right.

If the digit to the right is 0, 1, 2, 3 or 4, add 0 to the digit in the millionths place value. This is because the digits are nearest to 0.

If the digit to the right is 5, 6, 7, 8 or 9, add 1 to the digit in the millionths place value.

The rounded off answer must be to six decimal places.

Example 1

Round off 0.1550536 to the nearest millionths

Solution

0 . 1 5 5 0 5 3 6

6 is nearer to 1 millionth

Required place value

0.01550536 = 0 . 1 5 5 0 5 3 6 + 1 = 0 . 1 5 5 0 5 4

0.1550536 is rounded off to 0.155054.

Example 2

Round off 31.0938173 to the nearest millionths

Solution

3 1 . 0 9 3 8 1 7 3

3 is nearer to 0 millionth

Required place value

31.0938173 = 3 1 . 0 9 3 8 1 7 3 + 0 = 3 1 . 0 9 3 8 1 7

31.0938173 is rounded off to 31.093817.

Application activity 5.6

Round off the following to the nearest millionths

a) 0.1251076 b) 4.1407324 c) 0.0506985 d) 13.1303643 e) 1.1109427

5.7. Solving problems involving rounding off decimal numbers

Activity 5.7

Measure the length, then the width, of the football pitch (in cm).

Note the measurements on slips of paper.

What is the area in square meter to the nearest whole? (ones)

Round off the width in meters to the nearest tenths.

Discuss how rounding off can be applied in daily life.

Summary

To solve a problem,

- Read and understand the statements.
- Consider rounding off when estimating quantities if requested.
- If the digit in the place value to the right of the required place value is 0, 1, 2, 3, or 4, round down.
- If the digit in the place value to the right is 5, 6, 7, 8, or 9, round up.

Example

The area of a district is 11,033.1416 square kilometres.

a) Round off the area to the nearest tenths.

b) What is the area rounded off to the nearest thousandths?

Solution

(a) 11033.1416

1 1 0 3 3 . 1 4 1 6

4 is nearer to 0 tenths

Required place value

11,033.1416

\+ 0

11,033.1

Therefore, the area rounded to the nearest tenths is 11,033.1 square kilometres.

(b) 11033.1416

11033.1416

6 is nearer to 1 thousandth

Required place value

1 1 0 3 3 . 1 4 1 6

\+ 1

1 1 0 3 3 . 1 4 2

Therefore, the area rounded to the nearest thousandths is 11,033.142 square kilometres.

Application activity 5.7

a) A trader bought beans weighing 1,672.3 kg. Approximately how many kilograms were there when rounded to the nearest ones?

b) In a school store, there are 362.51 kg of beans and 506.78 kg of maize flour. Round off the total kilograms to the nearest tenths.

c) Michael Jordan ran 100 m in 9.83 seconds. Round off the time to the nearest seconds.

d) A forest has an area of 6.234 hectares. Round off this area to the nearest hundredths.

5.8. Converting fractions into decimals

Activity 5.8

Look at the fractions in part A and decimals in part B. Divide numerator by denominator and match results them accordingly. Explain your matching process to the whole class.

Part A	Part B
$\frac{6}{5}$	2.56
$\frac{64}{25}$	0.88
$5\frac{4}{10}$	1.2
$\frac{88}{100}$	5.4

Summary

When converting fractions into decimals, we divide the numerator by the denominator.

For mixed fractions like in example 3, first change the mixed fractions to an improper fraction, then divide the numerator by the denominator.

When a fraction gives a decimal number with repeating digits, the decimal number is called a recurring decimal. It is written with an ellipsis to the right.

Example 1

convert $\frac{5}{8}$ into decimals

Solution

$$
\begin{array}{r|l}
8 & 0.625 \\
 & 5000 \\
(5\times0)= & \underline{\;\;0\;\;\downarrow} \\
 & 50 \\
(6\times8)= & \underline{\;\;48\;\;\downarrow} \\
 & 20 \\
(2\times8)= & \underline{\;\;16\;\;\downarrow} \\
 & 40 \\
(5\times8)= & \underline{\;\;40\;\;} \\
 & 00
\end{array}
$$

Therefore, $\frac{5}{8} = 0.625$.

Example 2

convert $\frac{5}{4}$ into decimals

Solution

$$
\begin{array}{r|l}
4 & 1.25 \\
 & 500 \\
(1\times4)= & \underline{\;\;4\;\;\downarrow} \\
 & 10 \\
(2\times4)= & \underline{\;\;8\;\;\downarrow} \\
 & 20 \\
(5\times4)= & \underline{\;\;20\;\;} \\
 & 00
\end{array}
$$

Therefore, $\frac{5}{4} = 1.25$

Application activity 5.8

Convert the following fractions into decimals

a) $\frac{13}{20}$ b) $9\frac{2}{5}$ c) $\frac{1}{3}$ d) $\frac{5}{16}$ e) $2\frac{1}{10}$

5.9 Converting decimals into fractions

Activity 5.9

Write 3 decimal numbers:

The first with 1 decimal place.

The second with 2 decimal places.

The third with 3 decimal places.

Ignore the decimal point and write the denominator under each decimal, the zeros matching the number of decimal places.

What is your observation?

Summary

When converting decimals into fractions, simply ignore the decimal point and write all digits of decimal number as the numerator, then the denominator depends on the number of decimal place. To know the denominator, you count the number of digits to your right after the decimal places.

If the decimal place is 1, the denominator is 10.

If the decimal places are 2, the denominator is 100.

If the decimal places are 3, the denominator is 1,000...

Recurring decimals will be converted in S1.

Example 1

Convert 0.236 into a fraction and show your working

Solution

There are 3 decimal places, therefore divide by 1,000 and simplify where possible.

$$0.236 = \frac{0.236 \times 1000}{1000} = \frac{236}{1000} = \frac{113}{500}$$

Example 2

Convert 4.09 into a fraction and show your working.

Solution

There are 2 decimal places, therefore, divide by 100 and simplify where possible

$$4.09 = \frac{4.09 \times 100}{100} = \frac{409}{100} = 4\frac{9}{100}$$

Note that we will convert recurring decimals into fractions in S1.

Application activity 5.9

Convert the following decimals into fractions and simplify completely.

a) 0.1234 b) 19.67 c) 25.3

5.10 Solving problems involving converting decimals into fractions and fractions into decimals

Activity 5.10

Think of a decimal number with three decimal places.
Write it on a piece of paper.
Give it to your neighbour.
Ask him/her to round it to the tenths place value and convert it into a fraction.
Discuss the whole process.
Is the final result the same as the first? Explain.

Summary

When solving problems, read and try to understand the problem then choose how to solve it.

Example

Mugiraneza bought 2.35 m, 1.265 m and 0.75 m of cloth. What total length of cloth did Mugiraneza buy? Convert the answer to a fraction.

Solution

The total length of the cloth = 2.35 m + 1.265 m + 0.750 m = 4.365 m.

There are 3 decimal places, therefore, divide by 1000 and simplify where possible.

$$4.365 = \frac{4365}{1000} = 4\frac{73}{200}$$

Application activity 5.10

1. If 58 out of 100 textbooks in the school are mathematics, write a decimal for the portion of the school textbooks which are Mathematics.
2. A tailor wants to cut 24 small pieces of 0.4 m from a roll of cloth. What is the length of the roll of cloth as a fraction?
3. Kanyana cooked 0.25 kg of rice for lunch. What is the amount of the rice she cooked as a common fraction?

5.11. End unit assessment

Assessing knowledge and understanding:

1. Round off the following to the nearest underlined place value:
   (a) 12.<u>4</u>637 (b) 86.9<u>9</u>75 (c) 324.66<u>7</u>5 (d) 567.683<u>4</u>
   (e) 23.456<u>7</u>8 (f) 0.8462<u>4</u>347.
2. Define "place value" in your own words and give an example.
3. Convert 0.65 to a fraction.

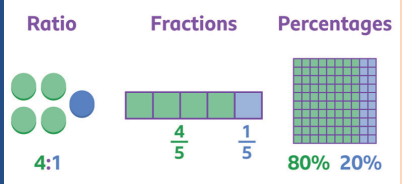
Assessing skills:

4. In a 100 m race, three runners used the following times: 9.85 s, 9.6 s and 9.625 s.
   a) What is the winning time?
   b) Express the winning time as a fraction.
   c) Write the winning time to 2 decimal places.
5. A cylinder has a 3.437 cm diameter. If it is increased to be 0.095 cm larger, what is the new size of that cylinder? Round off the answer to the nearest hundredths

Assessing attitudes and Values

6. Irankunda saves 1/4 of her earnings.
   a) Represent 1/4 as a decimal.
   b) Debate: "Is saving necessary?" Justify with two reasons.
7. A tailor cuts 1.875 m of cloth to make trousers but he needs 2 m.
   Is rounding up appropriate here? Why or why not?

UNIT 6



PERCENTAGES, RATIOS, AND RELATED PROBLEMS

Key Unit Competence: You will be able to work out percentages ratio and apply this knowledge to solve real life mathematical problem.

Learning objectives:

By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain the meaning and the role of percentages and ratios.

Skills:

- Convert percentages to decimals and vice versa.
- Share quantities in ratio.
- Apply percentages and ratios in solving mathematical problems.

Attitudes and values:

- Acknowledge the importance of percentages and ratios in daily life situations.
- Respect one another when working in groups to solve problems involving percentages and ratios.

6.0 Introduction

In this unit, you will learn how to compare quantities using ratios and express parts of a whole as percentages. You will explore real-life applications, such as understanding discounts, interpreting data, and solving problems involving proportions.

Ratios help us see relationships between numbers, while percentages make it easier to compare different quantities. Through interactive examples, practical exercises, and fun challenges, you will master these essential math skills.

Introductory Activity

Look at your classmates:

a) Find out the number of boys, then the number of girls. Can you express the number of boys or girls in terms of percentage for the total?

b) Can you share 36 Mathematics textbooks to the two groups (boys and girls) in your classroom in the ratio of the number of pupils per group?

c) Give other examples where the concepts of ratios and percentages are used in real life.

6.1 Converting percentages into decimals

Activity 6.1

You have been given marks in test and examinations.

Write the percentages on slips of paper, for example 75%.

Express the % mark as a denominator of 100.

Write each percentage value out of 100.

Considering the number of zeros in 100, write decimals with places of the same number.

What are your answers?

Make a class presentation.

Summary

Whenever you see a % symbol,

- Since "percent" means "per hundred", you can convert a percentage to a decimal by dividing it by 100. Example: 75% = $\frac{75}{100}$ = 0.75.
- A quicker way is to move the decimal point two places to the left.
  Example: Convert 20%: Move decimal left twice =0.20;
- Decimals are always expressed with a point in one of the digits after or before the digits of the whole number.
- Fractional percentages differ in the number of zeros, for example, $375\% = \frac{375}{100} = 3.75$.

Example 1

Convert $12\frac{1}{2}\%$ into decimals

Solution

First convert the percentage into fraction

$$12\frac{1}{2}\% = \frac{25}{2} \div 100 = \frac{25}{2} \times \frac{1}{100} = \frac{25}{200} = \frac{1}{8}$$

Change the fraction into a decimal

$$\begin{array}{r|l} & 0.125 \\ \hline 8 & 1 \\ -0 & \leftarrow 0 \times 8 \\ \hline 10 & \\ -8 & \leftarrow 1 \times 8 \\ \hline 20 & \\ -16 & \leftarrow 2 \times 8 \\ \hline 40 & \\ -40 & \leftarrow 5 \times 8 \\ \hline 00 & \end{array}$$

Therefore, $12\frac{1}{2}$ % is equal to 0.125.

Example 2

Convert 37.5% into decimals

Solution

First convert the percentage into fraction

$$37.5\% = \frac{375}{10} \div 100 = \frac{375}{10} \times \frac{1}{100} = \frac{375}{1000}$$

Three zeros denominators give 3 decimal places.

Therefore, 37.5% is equal to 0.375

Application activity 6.1

Convert the following percentages into decimals

(a) 48% (b) 97% (c) $45\frac{1}{2}\%$ (d) $20\frac{3}{4}$ (e) 56.6%

6.2 Converting decimals into percentages

Activity 6.2

Formulate five decimal numbers of 1, 2, and 3 decimal places.
Express them as fractions, but do not simplify.
Multiply each fraction by 100.
What are your answers?

Summary

To convert decimals into percentages, change the decimal into a fraction, then multiply the fraction by 100%.
Remember to put the percentage (%) mark to the right side.
To convert decimals into percentages, you can also move the decimal point two places to the right. This is because 100 has 2 zeros, hence 2 decimal places. Then add a % symbol.

Example 1

Express 0.35 as a percentage.

Solution

Method 1

Move the decimal point two places to the right:

$0.35 \rightarrow 3.5 \rightarrow 35$

Add a % sign: 35%

Therefore, 0.35 = 35%

Method 2

Multiply by 100%

$0.35 \times 100\% = \frac{35}{100} \times 100\% = 35\%$

Application activity 6.2

Convert the following decimal numbers into percentages

(a) 0.86 (b) 0.2 (c) 0.12 (d) 0.05

6.3 Converting percentages into fractions

Activity 6.3

Write four percentages on a slip of paper.

Express them as fractions of the denominator 100.

Simplify each fraction.

What do you get?

Summary

To change percentages into fractions, make the given figure the numerator and 100 the denominator, then reduce where possible.

To change fractional percentages into common fractions, divide the fractional part by 100. Then multiply by the reciprocal of 100.

Example 1

Change 40% into a fraction

Solution

First convert the percentage into a fraction

$$40\% = \frac{40}{100} = \frac{2}{5}$$

Example 2

change $33\frac{1}{3}\%$ into a fraction

Solution

Change into improper fraction

$$33\frac{1}{3} = \frac{100}{3}$$

Divide by the percentage

$$\frac{100}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

Application activity 6.3

Convert the following percentages into fractions:

(a) 20% (b) 50% (c) 12.5% (d) $\frac{1}{2}\%$

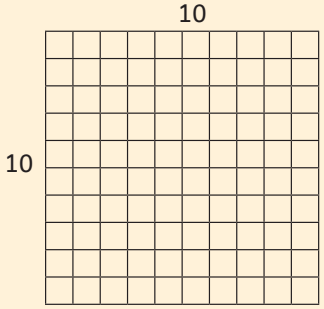
6.4 Converting fractions into percentages

Activity 6.4

Get four sheets of paper of 10 by 10 square grid like the one shown.

Study the fractions in the flash cards.

$\frac{1}{2}$ $\frac{9}{10}$ $\frac{79}{100}$ $\frac{7}{20}$



Present the fractions in the grids as follows:

$\frac{1}{2}$: shade 1 out of every 2 squares.

$\frac{9}{10}$: shade 9 out of every 10 squares.

$\frac{79}{100}$: shade 79 out of every 100 squares.

$\frac{7}{20}$: shade 7 out of every 20 squares.

Write the shaded squares out of the total squares in the grid.
What are your observations?

Summary

When converting a fraction into a percentage, multiply the fraction by 100%.

Then simplify to the simplest terms.

The percentage obtained is equivalent to the fraction.

Percentages are fractions with the denominator 100.

Example 1

convert $\frac{3}{4}$ into a percentage

Solution

Multiply the fraction by 100

$$\frac{3}{4}\times 100\% = 3\times 25\% = 75\%$$

Application activity 6.4

Change the following fractions into percentages

(a) $\frac{2}{5}$ (b) $\frac{21}{9}$ (c) $\frac{11}{2}$ (d) $\frac{1}{9}$

6.5. Comparing quantities as percentages

Activity 6.4

Convert the following into percentages:
6 pencils out of 1 dozen pencils.
30 minutes out of 1 hour.
200 grams out of 1 kilogram
250 metres out of 2 kilometres.
Present your working to the class.

Summary

When comparing quantities as percentages, first express the quantity as a fraction then multiply the fraction formed by 100%.

The total is always 100%. To find the original number, multiply the value of the unit percentage by 100%.

When comparing quantities as percentages, first convert the units to be the same. Express as a fraction, then multiply the fraction by 100%.

Example 1

Express 750 g as a percentage of 1 kg of sugar.

Solution

Express the given grams as a fraction a kilogram.

1 kg = 1,000 grams

$$750g = \frac{750}{1000}kg$$

multiply the fraction by 100%

$$\frac{750}{1000} \times 100 = 75\%$$

therefore, 750g of 1kg=75%

Example 2

45 out of 75 learners in P.6 got above the pass mark in a Mathematics examination. Which percentage of the class;

Passed?

Failed?

Solution

$$\text{percentage of learners who passed} = \frac{45}{75} \times 100\% = 60\%$$

number of learners that failed=75-45=30

$$\text{percentage of learners who failed} = \frac{30}{75} \times 100\% = 40\%$$

Or 100% - 60% = 40%

Percentage of the whole class (100%) - Percentage of pass (60%).

Application activity 6.5

1. What is the percentage of 36 minutes for 1 hour?
2. There are 80 girls in a school. If there are 400 learners in the whole school, what is the percentage of girls?
3. Express 500 g as a percentage of 2 kg of beans?
4. What percentage of 1 hour is:
   (a) 45 minutes? (b) 20 minutes?
5. A taxi covered 65 km of 130 km of the journey. What percentage of the journey was:
   (a) Covered? (b) Left?

6.6 Comparing percentages as quantities

Activity 6.6

Write 10% on slips of paper.

Let 10% represent 20 books.

Form groups of 10% equating them to group of 20 books. 10% is equivalent to 20 books.

20% is equivalent to 40 books.

30% is equivalent to 60 books.

40% is equivalent to 80 books and so on up to 100%

100% is equivalent to ? books.

100% is taken to be equivalent to total books.

How many books are there?

Present your working to the class.

Summary

Divide the given quantity by the given percentage to get the equivalent of 1%.

Multiply that fraction by 100% to get the total quantity/amount.

Example

5% of the learners in P 6 are boys.

If there are 20 boys in the class, how many learners are in the class?

Solution

5% of the total learners = 20 boys

Let the total number of learners be r.

5% of r = 20 boys

$$\frac{5}{100} \times r = 20 \leftrightarrow \frac{5r}{100} = 20 \leftrightarrow 5r = 2000 \leftrightarrow r = \frac{2000}{5}$$

r = 400, therefore, there are 400 learners in P6.

Application activity 6.6

1. 20% of a number is 60. What is the number?
2. 60 is equivalent to 10% of a number. Find the number.
3. $12\frac{1}{2}\%$ of a number is 1600. calculate the number ·
4. 45% of the fish in a pond are catfish. There are 900 catfish. How many fish are in the pond altogether?

5. 11% of the cars imported by RENEX Auto Mart every month are pickups.

If there are 55, find the total number of cars imported.

6.7 Increasing a Number by a Percentage

Activity 6.7

Dusabe had 1,000 Frw.

Her father increased it by 20%. How much does she have now?

20% increase means:

Each 100 Frw, the increase is 20 Frw. 200 Frw, the increase is 40 Frw. 300 Frw, the increase is 60 Frw.

....., the increase is ? Frw.

1,000 Frw, the increase is ? Frw.

Add the increase to 1,000 Frw.

What is your answer?

Summary

To increase a number by a given percentage, add the given percentage to 100%, then multiply by the given number.

To get the increased new amount, work out the increased amount, then add it to the old amount.

Increasing a given number by a given percentage means, adding the quantity on each 100 of the original quantity.

Example 1

Ancila's salary is 8,000 Frw per month.

How much money will her salary be if it is increased by 20%?

Solution

Method 1

Assume 100% to be equivalent to the old amount.

(100% + given %) of the old number

100% + 20%) of 8,000 Frw.

120% of 8,000 Frw.

$\frac{120 \times 8,000}{100}$ Frw = 120x80 = 9,600 Frw

Method 2

Calculate the increase first.

(Given % increase x old amount) + old amount

(20%) of 8,000 Frw

$\frac{20 \times 8,000}{100} = 1,600$ Frw

Add the increase to the old amount

1,600 Frw + 8,000 Frw

= 9,600 Frw.

Example 2

The price of peas was 2,400 Frw. It was increased by 20% in the first harvest season. In the second season it was increased by 10%. What is the new price?

Method 1

Assume 100% to be equivalent to the old amount.

(100% + given %) of the old price.

1st increase: (100% + 20%) = 120%

2nd increase: (100% + 10%) = 110%

New price

$\frac{120}{100} \times \frac{110}{100} \times 2400\text{Frw} = 12 \times 11 \times 24 = 3{,}168$

Therefore, the new price is 3,168Frw

Method 2

Calculate the increase first.

(Given % increase x old price) + old price

1st increase: (20% of 2,400)

$\frac{20 \times 2{,}400}{100} = 480\text{Frw}$

Add the increase to the old price

=480+2,400=2,880Frw

2nd increase (10% of 2,880)

$\frac{10 \times 2{,}880}{100} = 288\text{Frw}$

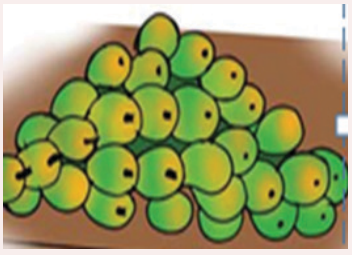
Add the increase to the old price.

=288+2,880=3,168Frw

Therefore, the new price is 3,168Frw

Application activity 6.7

1. Kamana had 50 mangoes. He purchased 30% more mangoes. How many did he get?



2. The population of a village was 600 people. If 40% migrated to the village area, how many people are in the village now?
3. A shirt was priced at 9,000 Frw last week. This week its price increased by 12%. What is the new price of the shirt?
4. The price of a pair of shoes is 40,000 Frw. This is increased by 15%.
   What is the new price of the shoes?

6.8 Decreasing a number by a percentage

Activity 6.8

If an item costs 500 Frw.Its price was decreased by 5%. What is new price now? Each 100 Frw, the decrease is 5 Frw.

200 Frw the decrease is 10 Frw.

..... Frw the decrease is .... Frw.

..... Frw the decrease is .... Frw.

..... Frw the decrease is .... Frw.

Subtract the total decrease from the original price.

What is the new price?

Summary

To decrease a number by a given percentage, subtract the given percentage from 100%, then multiply by the given number.

To get the new amount after decreasing, work out the decrease first, then

subtract it from the given amount.

Example 1

The 1,200 litres of fuel supplied to the Ministry of Education monthly was reduced by 25%. How much fuel is supplied to the ministry now?

Solution

Method 1

Assume the old amount to be 100%.
(100% – 25%) of old number.

= 75% of 1,200 litres of fuel

$$\frac{75}{100} \times 1,200 \text{litres} = 75 \times 12 = 900 \text{ litres of fuel}$$

Method 2

Calculate the decreased amount first

Old amount – (given % decrease × old amount)

= 25% of 1,200 litres

$$\frac{25}{100} \times 1,200 \text{frw} = 300 \text{litres}$$

(Old amount – decrease)

= 1200 – 300

= 900 litres of fuel

Example 2

The price of sugar decreased from 900 Frw by 10% then by 10%. What is the new price?

Solution

Method 1

Assume 100% to be equivalent to the old amount.

(100% - given %) of the old number.

1st decrease: (100% - 10%) = 90%

2nd increase: (100% - 10%) = 90%

$\frac{90}{100} \times \frac{90}{100} \times 900$Frw= 9x9x9 = 729 Frw

Method 2

Calculate the decrease first.

(Given % decrease x old amount) + old amount

1st decrease: (10% of 900)

$\frac{10 \times 900}{100} =$ 90 Frw

Subtract the decrease from the old amount.

=900 - 90 = 810 Frw

2nd decrease: (10% of 810)

$\frac{10 \times 810}{100} =$81 Frw

Subtract the decrease from the old amount.

= 810 - 81 = 729 Frw

Therefore, the new price is 729 Frw

Application activity 6.8

1. Decrease 1,500 by 24%.
2. 400 kg is decreased by 16%, what is the new amount?
3. The marked price of a new car was 4,000,000 Frw. It was reduced by 6%. At what price was it to be sold?
4. A trader bought 650 kg of beans at 600 Frw per kg. The price decreased by 8% after the harvest season. How much did she get after the sale?
5. Decrease 101,000 kg by 30% then by 10%.
6. The marked price of a motorcycle was 1,000,000 Frw. It was decreased by 12% then by 9%. What is the new price?

6.9 More about increasing and decreasing quantities by percentage

Activity 6.9

Think of an amount of money. Write it on slips of paper.

If the amount is increased by 40%, it becomes 28,000 Frw.

What was the old amount?

Explain your working to the class.

Summary

To find the unknown number, find 1% of the increased amount then multiply by 100.
To find the unknown number, find 1% of the decreased amount. Then multiply by 100.

Example 1

After increasing a number by 15%, it became 34,500. What is the number?

Solution

Assume the number to an unknown. Let the number be y.

Old number = (100% + 15%) of y

$$34,500 = \frac{115}{100}y \Leftrightarrow \frac{34,500}{1} = \frac{115y}{100}$$

$$115y = 3,450,00 \Leftrightarrow y = \frac{3,450,000}{115} = 30,000$$

Therefore, the old number is 30,000

Example 2

What amount when decreased by 12% becomes 528,000 Frw?

Solution

Let the number be x.

Old number = (100% - 12%) of x

528,000= 88% of x

$$528,000 = \frac{88}{100}x \Leftrightarrow \frac{528,000}{1} = \frac{88x}{100}$$

$$88x = 52,800,000 \Leftrightarrow x = \frac{52,800,000}{88} = 600,000 \text{Frw}$$

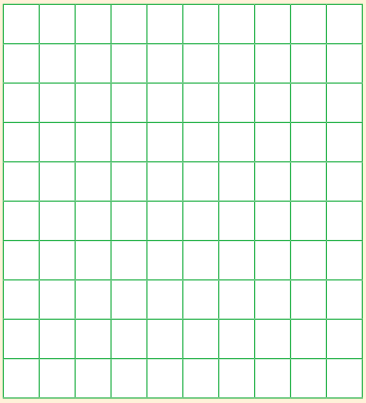
Application activity 6.9

1. What number when increased by 16% becomes 580?
2. After decreasing a certain amount of money by 9% it becomes 36,400 Frw. What was the amount?
3. Irebe harvested a certain amount of maize last season. This season the harvest increased by 6% to 848 kg. How many kilograms did he harvest last season?
4. What number when decreased by 20% becomes 1,600?
5. Akim's salary was increased to 540,000 Frw by 8%. Find his old salary.

6.10 Finding percentage increase and decrease

Activity 6.10

- Study the grid.
- Shade 40 squares.
- Then shade 60 squares including the first 40 squares with different colour.
- How many more squares have you shaded?
- Express the more shaded squares as a fraction of the first 40 squares.
- Convert the fraction into a percentage.
- What do you observe?
- Share your working out with the class.



Summary

$$\text{percenage increase} = \frac{\text{increase}}{\text{old number}} \times 100\%$$

$$\text{percentage decrease} = \frac{\text{decrease}}{\text{old number}} \times 100\%$$

Example 1

After increasing 600 by a certain percentage it becomes 840. Find the percentage increase.

Solution

Old number= 600

New number = 840

The increase = (840 - 600) = 240

$$\text{percentage increase} = \frac{\text{increase}}{\text{old number}} \times 100\%$$

$$= \frac{240}{600} \times 100 = 40\%$$

Example 2

When 48,000 Frw is decreased by x% it becomes 36,000 Frw. Calculate the value of x.

Solution

Old amount= 48,000 Frw New number = 36,000 Frw

The decrease = (48,000 - 36,000) Frw

= 12,000 Frw

$$\text{percentage decrease} = \frac{\text{decrease}}{\text{old number}} \times 100\%$$

$$= \frac{12,000}{46,000} \times 100\% = 25\%$$

Application activity 6.10

1. By what percentage will 900 be increased to become 1200?
2. When 2,000 is decreased by x% it becomes 1,600 Frw. Find the value of x.
3. The price of a shirt was increased by a percentage. The price increased from 6,000 to 8,000 Frw. Calculate the percentage increase.
4. The marked price of a car was 4,200,000 Frw. It was decreased by x% to become 4,000,000 Frw. What is the value of x?

6.11 Finding percentage profit and percentage los

Activity 6.11

Role-play buying and selling of items in a class shop.

In pairs, demonstrate when you gain more money and when you get less money than the amount you purchased the item.

Express the difference as a fraction of the buying price, then multiply by

100. How do you describe the result?

Discuss your answers with the class

Summary

Profit =selling price - cost price

Other terms used for profit are: gain, raise, increase and more.

$$\text{percentage profit} = \frac{\text{profit}}{\text{buying price}} \times 100\%$$

Loss = cost price - Selling price

Other terms used for loss are: reduction, less or decrease.

$$\text{Percentage loss} = \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

Example 1

A trader bought a bag at 10,000 Frw. He later sold it to a customer at 12,500 Frw. What was her percentage profit?

Solution

Buying price = 10,000 Frw Selling price = 12,500 Frw

Profit = Selling price – buying price

$$\text{percentage profit} = \frac{\text{profit}}{\text{buying price}} \times 100\%$$

$$= \frac{2,500}{10,000} \times 100\% = 25\%$$

Therefore, the percentage profit is 25%

Example 2

Muhakanizi bought a TV at 450,000 Frw. He made a loss after selling it at 300,000 Frw. Calculate the percentage loss.

Solution

Cost price = 450,000 Frw Selling price = 300,000 Frw

Loss = Cost price - selling price

$$\text{percentage loss} = \frac{\text{loss}}{\text{buying price}} \times 100\%$$

$$= \frac{150,000}{450,000} \times 100\% = \frac{10}{3} = 33\frac{1}{3}\%$$

Therefore, the percentage loss is $33\frac{1}{3}\%$

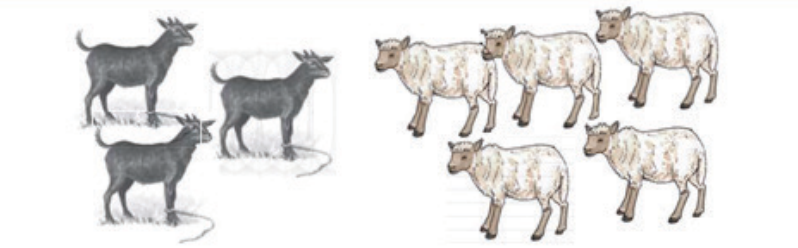
Application activity 6.11

1. Tereza sold a radio at 50,000 Frw. She had bought it at 40,000 Frw. Calculate the percentage profit.
2. Sales man made a loss after selling a dining table at 75,000 Frw which its original cost was 90,000 Frw. What was the percentage loss?
3. A grocer sold a pineapple at 500 Frw. She had purchased it at 400 Frw.Find the percentage profit.
4. Kwizera bought a school bag at 15,000 Frw. He realised that his books could not fit in it. He sold it at 12,000 Frw. What was the percentage loss?
5. A shopkeeper bought a sack of sugar at 750 Frw each kilogram. He soldevery kilogram at 800 Frw. Calculate the percentage profit.

6.12 Ratio and sharing quantities in ratios

Activity 6.12

In a farm, there are 3 goats to 5 sheep.



a) Do you know how you can present the ratio of the goats to sheep? Write it down.
b) How many sheep should be there if there are 6 goats in the same ratio?
c) Explain your working out

Summary

To formulate ratio; compare two quantities by division.

To share in reatio: Total parts a : b;

1. Add parts of the ratio: Total of parts = a+b
2. Find the value of 1 part = Total quantity ÷ (a+b)
3. Calculate share of each person:
   - a x value of one part
   - b x Value of one part

The sum of the shares is equal to the total quantiity

Example 1

Peter and Jane shared 25 sweets in ratio of 2:3. How many sweets did each get?

Solution

Add to find the total of the shares.

2 + 3 = 5 shares

Peter got 2 of 5 shares.

Jane got 3 out of 5 shares

Express as a fraction

Then multiply by 25 sweets

$$\text{peter got} = 25 \times \frac{2}{5} = 10 \text{ sweets}$$

$$\text{Jane got} = 25 \times \frac{3}{5} = 15 \text{sweets}$$

Example 2

Abdul and Sharifa contributed money to start a business in the ratio of 6:5 respectively. Sharifa contributed 450,000 Frw.

How much did Abdul contribute?

How much money was contributed?

Solution

Abdul: Sharifa

6:5

5 shares = 450,000 Frw

$$1 \text{ share} = \frac{450,000 \text{Frw}}{5} \times 1 = 90,000 \text{Frw}$$

$$6 \text{shares} = 90,000 \text{Frw} \times 6 = 540,000 \text{Frw}$$

Abdul contributed 540,000 Frw

Total shares = 6 + 5 = 11 shares

1 share = 90,000 Frw

11 shares = 90,000 × 11 = 990,000 Frw

Total amount contributed was 990,000 Frw.

Application activity 6.12

1. Share 500 in the ratio of 2:3.
2. Share 420 kg in the ratio of 1:5.
3. Kabanda and Mukamusoni shared 12,000 Frw in the ratio of 3:5. Find how much each got?
4. Two farmers shared 125 kg of beans to sow. How many kilograms did each get if they shared in a rati0 of 3:2?
5. The ratio of girls to boys in a primary school is 9:8. There are 450 girls in the school.
   a) Work out the number of boys.
   b) Find the total enrolment in the school.

6.13 Increasing and decreasing quantities in ratios

Activity 6.13

Get counters like beads or small stones.

Pick 30 of them and form groups of 5.

How many groups have your formed?

Now form a 7th group of the same number of counters.

How many counters are in all the 7 groups altogether?

How many more counters have you added to the 30 counters?

Discuss and make a class presentation.

Summary

$$\text{The increased/decreased amount by a ratio} = \frac{\text{new part}}{\text{old part}} \times \text{old quanity}.$$

In a ratio, the first part is the new part and the second is the old part.

Example 1

Increase 900 litres in the ratio of 3:2.

Solution

The new part is 3

The old part is 2

$$\text{new amount} = \frac{\text{new part}}{\text{old part}} \times \text{old quanity}$$

$$\text{new amaunt} = \frac{3}{2} \times 900l = 3 \times 450 = 1{,}350 \text{litres}$$

The new litres are 1,350 litres

Example 2

Decrease 1,400 Frw in the ratio of 4:7.

Solution

The new part is 4

The old part is 7

$$\text{new amount} = \frac{\text{new part}}{\text{old part}} \times \text{old quanity}$$

$$\text{new amaunt} = \frac{4}{7} \times 1{,}400\text{Frw} = 4 \times 200 = 800\text{Frw}$$

The new amount is 800 Frw

Application activity 6.13

1. Increase 720 in the ratio of 5:4.
2. Decrease 1,400 in the ratio of 7:10.
3. What amount do you get after increasing 21,000 Frw in a ratio of 9:7?
4. Find the amount you get after increasing 14,400 Frw in a ratio of 17:12.
   800 books were decreased in the ratio of 3:4. How many books are there?

6.14 Finding the ratio of increase and decrease

Activity 6.14

Write 400 on slips of paper. Increase it to 600.

Express the new number as a fraction of the old number.

Reduce it to the simplest terms, then write as a ratio.

What do you notice?

Summary

Ratio of increase = New amount to old amount. The new amount is bigger than the old amount.
Ratio of decrease = New amount to old amount. The new amount is smaller than the old amount.

Example 1

A school bought 2,000 kg of beans in term one. In term 2, it bought 2,500 kg. Find the ratio of increase.

Solution

$\text{new amount} = 2{,}500\text{kg}$

$\text{old amount} = 2{,}000\text{kg}$

$$\text{increse in ratio} = \frac{\text{new amount}}{\text{old amount}} = \frac{2{,}500}{2{,}000} = \frac{5}{4}$$

Therefore, the ratio of increase is 5:4

Example 2

Lucumu's salary was 45,000 Frw. It was decreased to 36,000 Frw. In what ratio was the salary decreased?

Solution

$\text{new salary} = 36{,}000\,\text{Frw}$

$\text{old salary} = 45{,}000\,\text{Frw}$

$$\text{decrease in ratio} = \frac{\text{new salary}}{\text{old salary}} = \frac{36{,}000\,\text{Frw}}{45{,}000\,\text{Frw}} = \frac{4}{5}$$

Therefore, the ratio of decrease is 4:5

Application activity 6.14

1. 500 was increased to 600. Find the ratio of increase.
2. 4,000 was reduced to 2,500. What is the ratio of decrease?
3. A trader used to purchase 144 cartons of markers. Now he purchases 168 cartons every week. Work out the ratio of increase.
4. The enrollment in a school dropped from 700 learners to 560 learners. What was the ratio of decrease?
5. What is the ratio increase from 108,000 to 156,000?

6.15 Solving problems involving ratios

Activity 6.15

Get 40 bottle tops.
Group them in twos and threes at the same time.
Write a statement about the above.
How many bottle tops are in each group?
Discuss, then make a presentation to class.

Summary

Read and understand the question for clear interpretation.

Writing in ratio is a simple way of comparing number.

Example1

Linda and Harry got sweets in the ratio of 3:4 respectively. If Harry got 12 sweets,

How many sweets did Linda get?

How many sweets did both share?

Solution

The ratio of Linda to Harry is 3:4

Harry got 12 sweets

let total number be x

$$\text{Harry got} \rightarrow \frac{3}{7} \times \text{total number} = 12 \text{ sweets}$$

$$\frac{4}{7} \times x = 12 \text{sweets} \leftrightarrow \frac{4x}{7} = 12 \leftrightarrow 4x = 84$$

$$x = \frac{84}{4} \leftrightarrow x = 21$$

ii. both got 21 sweets

$$\text{i. Linda got} = \frac{3}{7} \times 21 = 3 \times 3 = 9 \text{sweets}$$

Example 2

A class had 49 learners. The number decreased to 42 learners. In what ratio did the number decrease.

Solution

New number = 42

Old number = 49

$$\text{decrease in ratio} = \frac{\text{new number}}{\text{old number}} = \frac{24}{49} = \frac{6}{7}$$

$$= 6:7$$

Ratio of decrease = 6:7

Application activity 6.15

The cost of a pen and a pencil is in the ratio of 5:2. What is the cost of the pen if the cost of a pencil is 100 Frw.

There are 120 sheep on a farm. The ratio of ewes to rams is 5:7. How many rams are in the farm?

Alpha and Anne got 360 saplings from the forestry official. They shared them in the ratio of 4:5 respectively.

How many saplings did Alpha get?

How many saplings did Anne get?

The ratio of boys to girls in a school is 9:11. If there are 330 girls;

How many boys are in the school?

Find the number of learners in the school.

End of unit assessment of unit 6

Assessing knowledge:

1. Convert the following percentages into fractions

   (a) 20% (b) $5\frac{1}{2}$

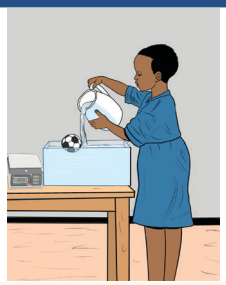
Assessing skills

2. Paul bought 6 blue pens. There were 50 pens in the whole packet. What is the percentage of blue pens in the whole packet?
3. $37\frac{1}{3}$% of the chicken mash prepared is made from silverfish. If this is 224 kg, what is the total mass of the mash prepared?
4. 28,000 Frw was increased by 10%, then by 10%. What did it become?
5. Decrease 800 by 12%.
6. What number when decreased by 12.5% becomes 1,050?
7. The price of a pair of shoes decreased from 20,000 Frw to 17,000 Frw/ Find the percentage decrease?
8. Mukandahiro bought a dozens of counter books at 1,500 Frw each. She later sold every book at 1,000 Frw. What was her percentage loss?
9. In our school, the ratio of teachers to learners is 1:8. If there are 10 teachers, how many learners are there?

Assessment of Attitudes and Values

10. A farmer's milk production decreased from 900L to 810L. Write the decrease as a ratio. Explain why maintaining production levels matters for food security.
11. Population increased from 3,200 to 4,800. Express as ratio. Discuss one positive and one negative impact of rapid population growth.
12. Market vendors often increase prices by 20% during shortages. Is this fair? Justify your answer.
13. Your friend says "Fractions are useless in real life." Give two examples from real life experience that prove otherwise.

UNIT 7



RELATIONSHIP BETWEEN VOLUME, CAPACITY AND MASS

Key unit competence: You will be able to compare units of volume, capacity, mass of water and apply this knowledge to solve real life problems.

Learning objectives:

By the end of this unit, you should be able to:

Knowledge and understanding:

- State units of length, capacity and mass in solving problems.
- Explain the relationship between volume, capacity and mass in the case of water.

Skills:

- Convert between units of volume, capacity and mass.
- Solve problems involving the relationship between volume, capacity and mass measurements.

Attitudes and values:

- Show respect to one another when working in groups.
- Appreciate the use of the relationship between units of mass, capacity and volume in real life.

7.0 Introduction

This unit deals with the conversion of units of capacity, length, mass and volume. It establishes the relationship between units of mass, capacity and units of volume of water.

In daily life, people use different containers (bottles, jerry cans, buckets, etc) to carry water or other liquids. Each container has a volume and a capacity and when it is full of water it has a certain mass. Therefore, we are going to study these 3 measurements.

Introductory Activity

a) Did you ever try to think about the relationship between the mass of water that fills the 1 litre bottle and the volume of that bottle in cubic decimetre?

Use a balance to measure the mass of 1ℓ of water.

b) Do you think that the volume of a container, its capacity and its mass may have a relationship? Explain if 1ℓ of water can fill a box of 1 cubic decimeter.

7.1 Revision on capacity measurements

Activity 7.1

- Get an empty jerrycan.
- Fill it with water.
- Pour water in one litre bottles.
- How many litre bottles are filled from one jerrycan?

Compare the capacity of a jerrycan with 1 litre bottle.

Summary

Capacity is the amount of liquid that fills a container.

The standard units of capacity are litres and milliliters: 1,000 mℓ = 1 ℓ

To convert one unit of capacity to the next smaller unit, multiply by 10.

Example 1

Convert: 3,450ℓ to kℓ.

Solution

1 kℓ = 1,000 ℓ

$3{,}450\ l = \frac{3{,}450}{1{,}000}$

$= 3.45\ kl$

Example 2

Work out: 15 kℓ 657 dℓ + 34 kℓ 23 dℓ = ...dℓ

Solution

$$\begin{array}{r} 15\ kl\ 657\ dl \\ +\ 34\ kl\ 023\ dl \\ \hline \boldsymbol{49\ kl\ 680\ dl} \end{array}$$

$15\ kl\ 657\ dl + 345\ kl\ 023\ dl = 490980\ dl$

Application activity 7.1

1. A tank of water contains 5 kℓ of water. How many jerrycans of 20 ℓ can be fully filled from that tank?
2. A container of 10 ℓ of juice was shared equally among 5 children, how many litres did each one get?
3. On a farm, 100 kℓ of milk are collected in a month. How many litres of milk are collected in 5 months if milk is produced at the same rate?
4. A jerrycan of 20 litres is poured into 3 litres small jerrycans.
   a) How many 3 litres small jerrycans are fully filled?
   b) How much water remains?

7.2 Revision on mass Measurements

Activity 7.2

- Take a stone outside of the classroom and measure its mass using a balance.
- Measure mass of different stones with different sizes and record the results in your notebook.
- Convert the results in different units: from kg to hg, dag, g, dg, cg and mg,
- Explain the meaning of mass and give an example.
- What do you notice?
- Present your findings to the class.

Summary

The **mass** of an object is the quantity of matter contained in the object.

The standard unit of mass is kg (kilogram) and g (grams).

For small quantities, g (gram) is used.

For large quantities, use kilograms.

To convert a mass from one unit to the next smaller unit, multiply by 10. Use a table of conversion or use the rule for direct proportion.

Example 1

Convert: 30 dag =_____g

Solution

1 dag = 10 g

30 dag = (30 × 10) g = 300 g

Example 2

Convert: 2,500 g = _____kg

Solution

1,000 g = 1 kg

$2,500g = \frac{2,500}{1,000} = 2.5\text{kg}$

Application activity 7.2

1. Work out and convert in the required units:
   a) 50 kg + 23 hg = _____dg
   b) 66 hg 56 g + 55 dg = _____ mg
2. Muziranenge bought 50 tonnes of beans. She sold 30 sacks each containing 80 kg. How many kg did she remain with?
3. Lydia bought the following items:1 kg of sugar, 5 kg of tomatoes, 250 g of salt, 400 g of tea bags. Find the total mass in kg of items she bought.
4. The following are the results of 5 learner's body mass tests: 57.4 kg; 30kg; 49.7 kg; 50.02 kg; and 44.3 kg. What is their total mass?

7.3 Measurement of volume

Activity 7.3

Find a box container, for example chalk box.

Measure in cm the length, width and height of the box with a ruler.

Write down the results.

- What is the volume of the box if:
  a) You measure in cm?
  b) Convert the result in mm^3.
- List the units of volume.
- Where is the measurement of volume useful in daily life? Explain your findings to the class.

Summary

Volume is measured in m^3. Other units of volume are: $km^3, hm^3, dam^3, dm^3, cm^3, mm^3$

Volume is the space occupied by an object. The volume determines how big or how small an object is.

Conversion of units of volume is done better using a conversion table.

To convert one unit of volume to the next smaller unit, multiply by 1000.

Example 1

Convert: 12 cm^3 to mm^3.

Solution

Place 12 in the first 2 places of cm^3

dm^3			cm^3			mm^3		
				1	2	0	0	0

Fill all the places to mm^3 with zeros (0).

Therefore 12 cm^3 = 12,000 mm^3.

Example 2

Convert: 3,400,000 cm^3 to m^3.

Solution

Place the number 34000000 in the table such that the o for ones is placed in the left cell of cm^3.

Place 4 in the last place of dm^3 and 3 in the first place of m^3.

Find all the places to cm^3 with zeros

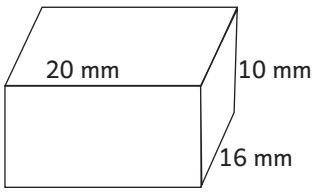
m^3			dm^3			cm^3		
		3	4	0	0	0	0	0

3,400,000 cm^3 = 3.4 m^3.

Example 3

A cuboid has a length, width and height of 20 mm, 16 mm and 10 mm respectively. Calculate its volume.

Solution



L= 20 mm; W = 16 mm H = 10 mm

Volume = Length × Width × Height

= 20 mm × 16 mm × 10 mm =

3,200 mm^3

Example 4

A rectangular tank has length, width and height of 2m, 1.5m and 3m respectively. Calculate its volume in cubic centimeters.

Solution

Length (l) = 2 m = (2 ×100) = 200 cm

Width (w) = 1.5 m = (1.5 × 100) = 150 cm

Height (h) = 3 m = (3 × 100) = 300 cm

Volume = Length × width × height

= 200 cm × 150 cm × 300 cm

= 9,000,000 cm^3

Application activity 7.3

1. With the help of the conversion table, work out the following:
   a) 15 dm^3 =_____ cm^3 b) 9,000,000 cm^3 =_____ m^3 c) 3 m^3 =_____ cm^3
2. Calculate the volume of cuboids with the following dimensions:
   a) 12 mm by 15 mm by 9 mm
   b) 15 cm by 8 cm by 10 cm
3. A rectangular water tank has a length of 4 m, width of 6 m and height of 2 m. Calculate its volume in cm^3.

7.4 Relationship between units of volume, capacity and mass

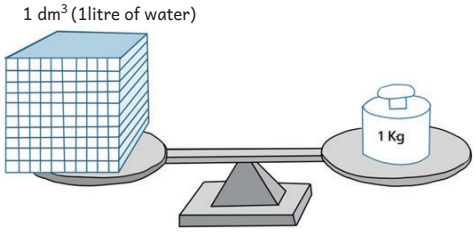
Activity 7.4

- Find a rectangular cup or tin of one litre and fill it with water.
- Measure the mass of water in the cup.
- Measure and calculate the volume of the cup.
- What is the relationship between these measurements?
- Explain your working steps to the class.

Summary

For water, volume, capacity and mass measurements are related by:

1 ℓ = 1 kg = 1 dm^3.



1 cm^3 (1 m*l*) of water = 1 g

1 dm^3 (1 *l*) of water = 1 kg

In the same way, 1m ℓ = 1 g = 1 cm^3.

This concept is helpful if you want to know the measurement of water when you already know its measurement in one of the above measurements.

Note: In this book, we can express liter by ℓ or by *l*

Example 1

What is the relationship between the following measurements? Litre (ℓ), kilogram(kg) and decimetre cubed (dm^3).

Solution

With the help of this conversion table,

m^3					dm^3	cm^3			mm^3		
		kl	hl	dal	l	dl	cl	ml			
		t	q	-	kg	hg	Dag	g	dg	cg	mg
					1						

The relationship is:

1 *l* = 1 kg = 1 dm^3

Example 2 : 20 kg = ...ml

Solution: 20kg= 20,000g = 20,000 ml

Application activity 7.4

1. Match the following units of measurements

Mass measurements	Volume measurements	Capacity measurements
1 *kg*	1 cm^3	1 *ml*
1 *g*	1 m^3	1 *dl*
1 t	1 dm^3	1 *kl*
1 *hg*	100 cm^3	1 *l*

2. Find out how many litres are in a 20 kg of water.
3. Does 1 litre of water equal 1 kg? Explain.
4. Why is the relationship between these measurements applicable when measuring water?

7.5 Real life problems involving the conversion between units of volume, capacity and mass of water

Activity 7.5

Convert the following quantity of water in respective units:

1) 10 kg = ... k ℓ = ... dm^3

2) 1 dm^3 = ... m ℓ = ... dg

3) 13 ℓ = ...hg

4) 78 h ℓ = ...kg

What do you notice?

m^3			dm^3			cm^3			mm^3		
		kl	hl	dal	l	dl	cl	ml			
		t	q	-	kg	hg	dag	g	dg	cg	mg
		7	8	10	30	0					

Summary

When converting capacity, volume and mass, use a conversion table including all the units.

m^3			dm^3			cm^3			mm^3		
		kl	hl	dal	l	dl	cl	ml			
		t	q	-	kg	hg	dag	g	dg	cg	mg

Example

Convert: (a) 13 dm^3 to hg (b) 7,800 ℓ to m^3

Solution

m^3			dm^3			cm^3			mm^3		
		kl	hl	dal	l	dl	cl	ml			
		t	q	-	kg	hg	dag	g	dg	cg	mg
		7	8	10	30	0					

Therefore, 13 dm^3 = 130 hg; 7,800 ℓ = 7.8 m^3

Application activity 7.5

1. Convert the following:
   (a) 6,767 kg = _____cℓ (c) 469 dl to mℓ (e) 4 m^3 = ____g
   (b) 1.7m^3 to cm^3 (d) 9,865 ℓ = _____t (f) 3.25 mg to g
2. Jacques fetches water with a jerrycan of twenty litres twice a day. How many litres does he fetch through the whole day? How many kilograms of water does he fetch?
3. A rectangular water tank has a capacity of 10,000 litres.
   (a) Calculate its volume in dm^3 . (b) What is the mass of water in tonnes?

End of unit assessment of unit 7

Assessing knowledge and understanding:

1. a) Fill in the gaps: (i) 777 dℓ = _____ kg (ii) 589,000 g = _____ m^3

   (b) Convert in given units: (i) 469 dℓ to mℓ (ii) 1.7 m^3 to cm^3.

2. Which quantity is more: 1 kg of water and 1 kℓ of water? Explain.

Assisiging skills

3. 800 jerrycans fill a tank. One jerrycan weighs 20 kg. How many litres does the tank hold?

4. A bucket contains 45.6 litres of water. The empty bucket weighs 3 kg.
   i) What is the mass of the full bucket?
   ii) What is the volume of the water that fills the bucket in cm^3?

5. There are 500 learners in a school. Lunch time each learner takes one cup of tea. If the cup holds 50 cℓ, what is the volume of tea taken throughout the whole school in cm^3?

6. 5 jerrycans full of water weigh109 kg. Each of the empty jerrycansweighs 800g
   a) Find the capacity of the water in the jerrycans.
   b) If Kalisa uses 2ℓ of that water, what is the mass of the water that will remain in kilograms?

7. The mass of water a school uses daily is 1.4 tonnes:
   a) Find the daily capacity of water the school uses in litres.
   b) Calculate the capacity of water it uses in a week (excluding the weekend) in decalitres

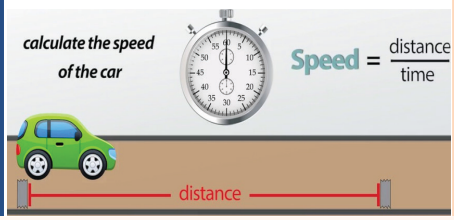
8. A family uses 15 ℓ of milk per week. How many 250 mℓ bottles is this?" (Convert ℓ to mℓ and divide).

Assessing attitudes and values

9. Why is it important to know the volume of water your household uses daily? How can this help the environment?"

10. "If two villages share a water tank, how would you divide 5,000 ℓ fairly if one village has 300 people and the other has 200 people?"

UNIT 8



SPEED, DISTANCE AND TIME

Key Unit Competence: You will be able solve problems related to different time zones, calculate and convert speed between km/h and m/s and vice versa.

Learning objectives:

By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain the relationship between a 12 hour clock and a 24 hour clock.
- Define speed, distance and time.
- Identify different units of speed, distance and time.
- Explain what determines time zones.

Skills:

- Tell the time using a 12hour clock and a 24 hour clock.
- Convert from km/h to m/sec and vice versa.
- Solve simple problems involving the calculation of speed, distance and time in real life situations.
- Solve problems that relate to different time zones.

Attitudes and values:

- Appreciate the importance of time management in daily life situations.
- Show concern towards respecting one another in group activities and welcoming group ideas.
- Appreciate the relationship between speed, distance and time to understand the time management.

8.0 Introduction

In this unit, you will learn how speed, distance, and time are related in mathematics and everyday life. Speed tells us how fast something moves, distance measures how far it travels, and time tracks how long the journey takes. By understanding the simple formula Speed = Distance ÷ Time, you can solve real-world problems, such as calculating how long a trip will last or determining the speed of a moving car.

You will also practice converting between different units, like kilometers per hour (km/h) and meters per second (m/s). Mastering these concepts will help you plan journeys, compare travel options, and apply math to some situations like sports and transportation of goods in vehicles.

Introductory Activity

When a business person must deliver goods to a customer,

a) Should the business person travel faster or slower? By which means?

b) What might be the result for late delivery of goods to the customer?

c) Do you think that you can calculate the time used when you have the speed of a vehicle, and the distance travelled? If yes, how can you do it?

8.1 Comparing the 12-hour format to the 24-hour format

Activity 8.1

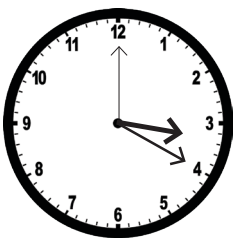
- You have learned about telling time.
- Choose any time in the morning and in the afternoon.
- Tell time using the 12-hour and 24-hour formats.
- Show time charts for 12-hour and 24-hour format.
- Compare the following time and explain what it means.

Summary

- The 12-hour format starts immediately after midnight until 12.00 noon, then from 12:00 noon to 12:00 midnight.
- The 24-hour clock starts immediately after 24:00 hours, then back to 24:00 hrs.
- To change from 12-hour clock to 24-hour clock, add 12.00 hours to the given time if it is post meridiem or past mid-day (P.M).
- To change from 12-hour clock to 24-hour clock, add 00.00 hours to the given time if it is ante meridiem or before mid-day (A.M).

Example

The clock shows that we are in the afternoon.



What time is it? Change the time to the 24-hour clock format.

Solution

It is twenty minutes past three

Therefore, we write 3:20 P.M in the 12-hour format. To change 3:20 P.M to the 24-hour clock, add 12:00 because 3 hours 20

Therefore, 3:20 p.m will become;

$$\begin{array}{r} 03:20 \\ +\ 12:00 \\ \hline \mathbf{15:20} \end{array}$$

Therefore, 3:20 P.M will become 15:20 in the 24-hour clock.

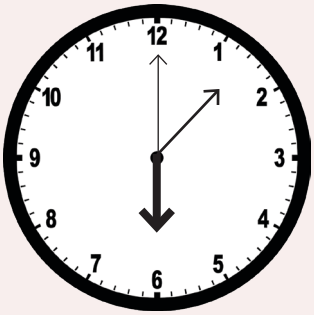
Application activity 8.1

1. Compare the 12-hour clock in the 1st column with the 24-hour clock in the 2nd column. Then match each time in column to its respective 24- hour clock in the 2nd column.

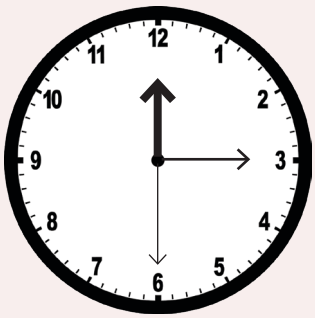
1st column	2nd column
3 :23 p.m.	06:45
6:45 a.m.	20:44
7:15 a.m.	15:23
12:12 p.m.	07:15
8:44 p.m.	12:12

2. Indicate the time in form of 12-hours and then in form of 24-hours.

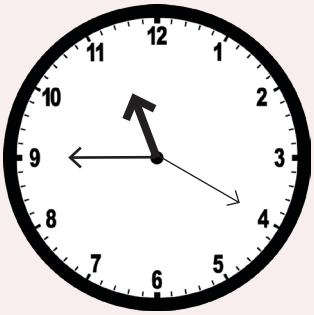
a) It is in the afternoon



b) It is in the morning



c) It is in the morning



d) It is in the evening

3. Change the time from 12-hour format to 24-hour format

(a) 4:21 P.M (b) 5:56 P.M (c) 9:12 A.M. (d) 8:45 A.M (e) 12:46 A.M.

8.2 Converting 12-hr format to 24-hr format and vice versa

Activity 8.2

Match the time in 24-hour format with that in 12-hour format.

24-hour	12-hour
12:45	11:30 p.m
07:30	12:45 p.m
23.30	9:15 p.m
21:15	4:20 a.m
04:20	7:30 a.m

What rule to follow when converting 24-hour format to 12-hour format, time above 12:00 hours?

Summary

To change a.m. time to the 24 hours format, add 00:00 hours.

To change p.m. time to the 24 hours format, add 12:00 hours.

To convert 24-hour format to the 12 hours format, time below 12:00 PM always add 00:00 hours. If the hour is 00, replace it with 12 and keep minutes the same.

To convert 24-hour format to 12-hour format, time above 12:00 hours always subtract 12:00 hours.

12:00 A.M to 12:59 noon does not change. It remains as it is.

24-hours = 00:00 A.M or 12:00 midnight, 00:40 = 12:40 A.M,12:30 P.M = 12:30.

Time above 12:00, that is 12:00 to 24:00, is always P.M.

Time below 12:00, that is 00:01 to 11:59 is always a.m.

12:00= 12:00 midday or 12:00 noon.

Example 1

Convert 6:24 A.M to 24-hr format.

Solution

A.M time changes directly to 24-hour format, so add 00:00 hours.

$$\begin{array}{rr} & 06:24 \\ + & 00:00 \\ \hline & \mathbf{06:24} \end{array}$$

Therefore, 6:24 A.M = 06:24.

Example 2

Convert 11:28 P.M to 24-hr format.

Solution

P.M time changes to 24-hour format by adding 12:00 hours.

$$\begin{array}{rr} & 11:28 \\ + & 12:00 \\ \hline & \mathbf{23:28} \end{array}$$

Therefore, 11:28 P.M = 23:28.

Example 3

Change 06:46 to 12-hour format.

Solution

The time begins with 0 so it is A.M.

Time below 12:00 subtract 00:00.

06:46 – 00:00 = 6:46 A.M.

$$\begin{array}{rr} & 06{:}46 \\ - & 00{:}00 \\ \hline & \mathbf{06{:}46} \end{array}$$

Time from 12:00 midnight to 12:59 A.M remains unchanged.

Therefore, 06:46 = 6:46 A.M

Example 4

Change 00:25 to 12-hour format.

Solution

The time begins with 0 so it is A.M time.

Time after midnight, add 12:00.

00:25 + 12:00 = 12:25A.M.

$$\begin{array}{rr} & 00{:}25 \\ + & 12{:}00 \\ \hline & \mathbf{12{:}25} \end{array}$$

Time from 12:00 midnight to 12:59 AM continues unchanged.

Therefore, 00:25 = 12:25A.M.

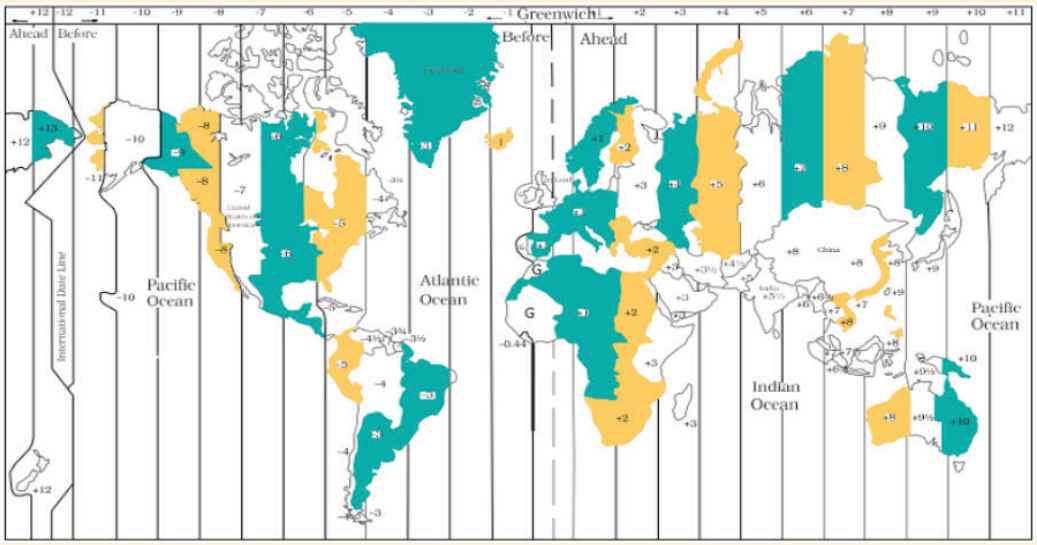
Application activity 8.2

1. Change the time from the 12-hour format to the 24-hour format.
   (a) 4:21 P.M (b) 8:45 A.M (c) 12:46 A.M.
2. Change the following time from 24-hour format to 12-hour format.
   (a) 04:12 (b) 15:54 (c) 06:32 (d) 22:10

8.3 The concept of time zones

Activity 8.3

1. Do you think that Kinshasa and other Places that are on West of your country's time zone are earlier in time? When it is 2:00 pm at Kigali, is Kinshasa earlier?



2. Do you think that Somalia and other Places that are East of your country's time zone are further in time?
3. Look at the world map showing standard time zones and answer the following:
   a) In how many time zones is the world divided?
   b) How many hours does each zone represent?
   c) Locate Rwanda on the world map.
   d) What do you observe above time of places left of Rwanda?
   e) What about time of countries to Rwanda's right?

Summary

A time zone is a region of the globe that observes a uniform standard time, for legal, commercial and social purposes.

The world is divided into 24 time zones.

Each time zone represents 1hour difference in time.

Moving to the West we reduce time.

Moving to the East, we add time. There is one hour per time zone.

Remember that between two longitudes (time zones) there is 15° which is equal to 1 hour.

Example 1

Look at the world map showing the standard time zones. What is the time in the 9th time zone towards the West?

Solution

9th time zone to the West is represented by -9. The time reading is 3:00 A.M

Example 2

Look at the world map showing the standard time zones. Find the time in the 11th time zone towards the East

Solution

11th time zone to the East is represented by +11. The time reading is 23:00 hrs or 11:00 P.M.

Application activity 8.3

Use the world map showing standard time zones to answer the following questions.

If it is 12:00 am at UTC, find the time reading in the time zones below:

(a) 2nd time zone to the East.
(b) 10th time zone to the East.
(c) 9th time zone to the East
(b) 7th time zone to the West.
(d) 12th time zone to the West.
(f) 5th time zone to the West

8.4 Solving mathematical problems relating to time zones

Activity 8.4

Study the map of the world showing the standard time zones.

- Name the meridian that is representing 0 degrees.
- How important is the meridian that you named?
- What important line is represented by -12 or +12?
- Give the importance of that line.

Summary

- To find time of another time zone to the West, multiply its position from the time in the given time zone by 1 hour, then subtract from the given time.
- To find time of another time zone to the East, multiply its position from the time in the given time zone by 1 hour, then add to the given time.
- In all calculations, use the 24-hour format to make calculations easier.
- There is a difference of one hour between two time zones.

Example 1

It is 11:30 P.M. in Rwanda. What time is it in New York which is in the 7th time zone to the West of Rwanda?

Solution

Places to the West are earlier in time.

The places in the first time zone to the left is 1 hour earlier.

New York is in the 7th time zone from Rwanda.

1st time zone = 1 hour x 7th time zone

= 7 x 1 hour = 7 hours.

Subtract 7 hours from the time in Rwanda

$$\begin{array}{rr} & 11{:}30 \\ - & 7{:}00 \\ \hline & \mathbf{4{:}30} \end{array}$$

Therefore, the time in New York is 4:30 P.M.

Example 2

The time in a certain time zone is 3:45 P.M. What is the time in the 6th time zone to the East?

Solution

Places to the East are further in time.

The places in the first time zone to the right is 1 hour less.

Read off the time in the 6th zone to the East.

1st time zone = 1 hour 6th time zone

= 6 x 1 hour = 6 hours.

Add 6 hours to the time given

$$\begin{array}{rr} & 3{:}45 \\ + & 6{:}00 \\ \hline & \mathbf{9{:}45} \end{array}$$

Therefore, the time is 9:45 P.M

Application activity 8.4

Work out the following:

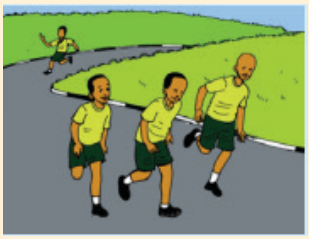
1. It is 2:00 P.M. in Cairo. What is the time in Delhi which is in the 3rd time zone to the East of Delhi?
2. The time in Accra is midnight. What time is it in Buenos Aires, in the 4th time zone to the West?
3. It is 12:00 noon at the Greenwich meridian. What time is it at the International Date Line?
4. The time in Sydney is 10:00 A.M. Baghdad is in the 7th time zone to the West of Sydney. What is the time in Baghdad?

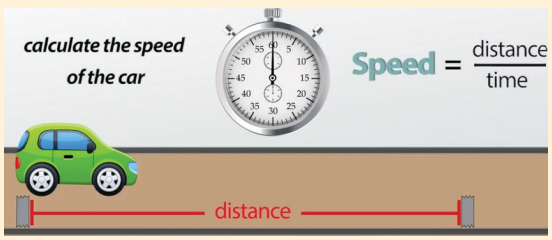
8.5 Speed of a moving body

Activity 8.5

Take a stop clock.

Start it and start running around the field at the same time.





Stop it when you return to the starting point.

Count the time you have taken.

Divide the distance with the time you took.

What do you notice?

Summary

- Speed is the distance travelled by an object in a unit of time.
- The standard international unit speed is metre per second (m/sec) and the everyday unit is kilometre per hour (km/h).
- To calculate speed, take the distance covered divided by the time taken.

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

- Speed determines how fast or how slow an object is moving.
- To find the distance covered, take the speed times the time used.

$$\text{Distance} = \text{Speed} \times \text{time}.$$

Example 1

A motor cyclist travelled for 3 hours and covered a distance of 210 kilometres. What speed was he moving?

Solution

Distance = 210 km, Time = 3 hours.

To calculate speed, divide the distance covered by the time taken to cover the distance.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{210\text{km}}{3\text{h}} = 70\text{km/h}$$

The speed of the motorcyclist was 70 km/hr.

Example 2

An athlete ran 600 m in 1 minute. Calculate his speed in m/sec.

Solution

Distance = 600 m, Time = 60 sec.

To calculate speed, divide the distance covered by the time taken to cover the distance.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{600\text{m}}{60\,\text{sec}} = 10\text{m/sec}$$

The athlete ran at a speed of 10 m/sec

Application activity 8.5

Work out the following:

1. A bus travelled for 3 hours to cover a distance of 180 km. At what speed was the bus moving?
2. At what speed was the car travelling if it covered 540 km in 6 hours?
3. What is the speed, when the distance and time are:
   a) 160 km and 4 hours?
   b) 150 km and 3 hours?
   c) 200 km and 5 hours?

8.6 Converting the speed from km/h to m/sec

Activity 8.6

Muziranenge ran 6 km in one hour.

If she was running at the same rate, can you find the distance in metres she covered in one second?

Discuss and defend your answer by showing your working steps.

Summary

1 hour = 60 minutes.

1 minute = 60 seconds.

1 hour = 60 x 60 seconds = 3,600 seconds.

1 km = 1,000 m.

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}$$

To change the speed from km/hour to m/s, use the following:

Covenvert km to m and hours to seconds, then devide:

$$\frac{\text{speed} \times 1{,}000\text{m}}{3{,}600\,\text{sec}}$$

Example 1

Express 72 km/h as m/sec.

Solution

Change km to m and hr to sec

$1\text{km} = 1{,}000\text{m}$

$1\text{h} = 60\text{min}$

$1\text{min} = 60\text{sec}$

$1\text{h} = 1 \times 60 \times 60\text{sec}$

$1\text{h} = 3{,}600\text{sec}$

$$72\text{km/h} = \frac{72 \times 1{,}000m}{1 \times 3{,}600\,\text{sec}} = 20m/\text{sec}.$$

Example 2

Convert 108 km/h into m/sec.

Solution

Change km to m and h to sec

$1\text{km} = 1{,}000\text{m}$

$1\text{hr} = 60\text{min}$

$1\text{min} = 60\text{sec}$

$1\text{h} = 1 \times 60 \times 60\text{sec}$

$1\text{h} = 3{,}600\text{sec}$

$$108\text{km/h} = \frac{108 \times 1{,}000m}{1 \times 3{,}600\,\text{sec}} = 30m/\text{sec}.$$

Application activity 8.6

a) Change the following km/h into m/sec:

b) (a) 90 km/h (b) 60 km/h (c) 180 km/h (d) 54km/h

c) The distance from village A to village B is 720 km. A car takes 6 hours to cover the journey. Calculate its speed in m/sec.

d) Jane covered a distance of 50 km in 2 hours.

e) Calculate the speed in km/h.

f) What was the speed in m/sec?

8.7 Converting the speed from m/sec to km/h.

Activity 8.7

During breaktime, learners of P6 had a 100 m race.

The winner covered 5 metres in one second.

Suppose he runs at the **same rate,** what distance can he cover in one hour?

Explain your working out.

Summary

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}$$

1,000 metres = 1 kilometre.

3,600 seconds = 1 hour.

To change m/sec to km/h, change m to km and sec to hrs, then divide.

Example 1

Express 200 m/sec as km/h.

Solution

Change m into km and sec to h.

1 km = 1,000 m

$$\text{so } 200\text{m} = \frac{200}{1,000}\text{km}$$

1 hr = 60 minutes

1 min = 60 sec

1 hr = 1 x 60 x 60 sec

1 hr = 3600 sec

$$\text{so } 1\text{sec} = \frac{1}{3,600}\text{h}$$

$$200\text{m/sec} = \frac{200}{1,000} \div \frac{1}{3,600}$$

$$= \frac{200}{1,000} \times \frac{3,600}{1} = \frac{20 \times 36}{1 \times 1} = 720 km/h$$

Example 2

Express 45 m/sec as km/h.

Solution

Change m into km and sec to h.

1 km = 1,000 m

$$\text{so } 45\text{m} = \frac{45}{1,000}\text{km}$$

1 hr= 60 minutes

1 min = 60 sec

1 hr = 1 x 60 x 60 sec

1 hr = 3600 sec

$$\text{so } 1\text{sec} = \frac{1}{3,600}\text{h}$$

$$45\text{m/sec} = \frac{45}{1,000} \div \frac{1}{3,600}$$

$$= \frac{45}{1,000} \times \frac{3,600}{1} = \frac{45 \times 3.6}{1 \times 1} = 162 km/h$$

Application activity 8.7

1. Convert the following into km/hr:

   (a) 15 m/sec (b) 45 m/sec (c) 25 m/sec (d) 100 m/sec

2. An aeroplane covered 1,000 metres in 5 seconds. Find its peed and express the speed in km/h.

3. During a bicycle Rwanda Tour Racing, a cyclist covered 100 metres in 10 seconds. Calculate the constant speed in kilometres per hour.

4. In a school competition, one athlete ran 100 metres in 15 seconds. What was the constant speed in kilometres per hour?

8.8 Distance covered by a moving body

Activity 8.8

- Mugenzi and Mutoni had a journey from home to visit a friend called James.
- They took 2 hours moving at a speed of 13 kilometres per hour to reach there.
- What is the distance from their home to James' home?

Summary

Distance = speed × time.

If departure and arrival time is given, calculate the duration to get the time.

Distance is measured in kilometres (km), hectometres (hm), decimetres (dam), metres (m), centimetres (cm) and millimetres (mm).

The standard unit of measuring distance is metre (m).

Example 1

A bus left Kigali at 10:00A.M travelling at 60 km/h. It arrived at its destination at 1:00P.M. Find the distance it covered.

Solution

Distance = Speed x Time

Time taken = 1:00P.M to 10:00 A.M

The duration crossed from morning to afternoon

Time = (12.00 - 10.00) + 1.00

= 2 hrs + 1 h.

= 3 hours.

Speed = 60km/h

Distance = 60 km/h x 3hrs = 180 km

The distance covered is 180 km.

Example 2

A car took 4 hours to cover a distance. It moved at a speed of 90 km/hr. Calculate the distance it covered.

Solution

Speed = 90 km/h

Time = 4 hours

Distance = speed × time

Distance = 90 km/h x 4hrs=360 km

The distance covered is 360 km.

Application activity 8.8

1. It takes Mugeni 2 hours walking at a speed of 5 km/hour to arrive at the market. What is the distance between her home and the market?
2. Teacher Mugenzi rode his bicycle for 2 hours at a speed of 12 km/h to arrive at school. What is the distance from his residence to the school?
3. Find the distance that Muhire covers in 5 hours moving at a speed of 80 km/h.
4. What is the distance covered by Therese in 15 seconds, if she moves at a speed of 72km/hour in her car?

8.9 Time taken by a moving body to cover a certain distance

Activity 8.9

Joseph walks to school every day.

The distance from home to school is 3 km.

One day, he walked at a speed of 2km/h. How long did he take to reach school?

Present your working out to the class.

Summary

$$\text{Time} = \frac{\text{distance covered}}{\text{speed}}$$

Time taken is worked out by dividing the distance covered by the speed.

The units of time are hours, minutes and seconds.

Example 1

Juliet walked a distance of 45 km at a speed of 5 km/h. What time did she take?

Solution

Distance = 45 km, Speed = 5 km/h.

$$\text{time} = \frac{\text{distance covered}}{\text{speed}}$$

$$= \frac{45\text{km}}{5\text{km/h}} = 9\text{h}$$

Express the time in the units given in the question. Therefore, she took 9 hours.

Example 2

Mutesi drove a racing car for a distance of 8,000 metre at a speed of 250 m/sec. What time did she take?

Solution

Distance=8,000m, speed=250m/sec

$$\text{time} = \frac{\text{distance covered}}{\text{speed}}$$

$$= \frac{8,000\text{m}}{250\text{m/sec}} = 32\,\text{sec}$$

Express the time in the units given in the question. Therefore, she took 32 sec.

Application activity 8. 9

Work out the following:

1. Christine drives her car at a speed of 72 km/h. What time does she take for the whole journey if she drives to cover a distance of 216 km?
2. A bus travelled from Kigali to Kampala. It was moving at a constant speed of 60 km/h. It stopped after moving 180 km. What time did it take to cover the journey?
3. Murengezi walks at 2 km/h when he is going to school. If he moves a distance of 3 km, how long does it take him to arrive at school?
4. How long did a car travelling at 60 km/h take to cover 240 km?

8.10 Calculating average speed

Activity 8.10

The distance from town A to town B is 120 km.

A bus covers the distance in 2 hours. It continues to town C a distance of 180 km from town B.

The bus takes $2\frac{1}{2}$ hours to cover the distance.

a) Find the total distance.

b) What was the total time taken?

c) Divide the total distance covered by the total time taken. What do you get?

Make a class presentation

Summary

Average speed is distance covered per unit time by an object moving, assuming it moved at a constant speed.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

Average speed can also be calculated for two or more distances covered within different time taken.

Example 1

A taxi moved from town P to town Q, a distance of 160 km. It took $2\frac{1}{2}$ hours. From town Q to town R, it covered 200 km in $3\frac{1}{2}$ hours. Find the average speed for the whole journey.

Solution

Find the total distance covered (TDC)

P to Q to R =160km + 200km = 360 km

Find the total time taken (TTT)

$$\text{P to Q to R} = 2\frac{1}{2} + 3\frac{1}{2} = 6\text{hours}$$

$$\text{average speed} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{360\text{km}}{60\text{h}} = 60\text{km/h}$$

The average speed is 60km/h

Example 2

A car takes 6 hours to cover a journey at 70 km/h. It takes only 4 hours to return through the same distance. Calculate the average speed for the whole journey.

Solution

i) Going Distance
= Speed ×Time.
Speed = 70 km/h.
Time = 6 hours.
Distance = 70 × 6 = 420 km

ii) Returning
Distance = 420 km
Time = 4 hours.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$= \frac{(420+420)\text{km}}{(6+4)\text{h}} = \frac{840\text{km}}{10\text{h}} = 84km/h$$

The average speed is 84km/h.

Application activity 8. 10

1. Town A is 40 km from Town B. A bus took 1 hour to cover the distance. The bus continued from Town A to another town 60 km away taking $1\frac{1}{2}$ hrs. What was its average speed?
2. A lorry takes 4 hours to travel from town (A) to town (B) covering a distance of 180 km. It uses 2 hours to return. Calculate the average speed.
3. A bus takes 6 hours to cover a distance of 480 km. It returns using only 4 hours. What was its average speed?
4. A motorcycle travelled from Kigali to Karongi covering a distance of 63 km. It took 1 hour 40 minutes. It returned through the same route covering the distance in 1 hour 20 minutes. What was the biker's average speed?

8.5 End unit assessment

Assessing knowledge and understanding

1. Define **speed** and write its formula.
2. Convert **5 hours** into seconds.
3. If a car travels **180 km in 3 hours**, what is its speed in km/h?
4. How many **time zones** are there in the world?
5. If it is **12:00 PM** in Time Zone **0 (GMT)**, what time is it in **Time Zone +3**?

Assessing skills

6. The distance from town A to town B is 720 km. A car takes 8 hours to cover the journey. Calculate its speed in m/sec.
7. A bus takes 4 hours to cover a journey of 240 km and takes only 2 hours to return. Calculate the average speed for the whole journey.
8. It is 8:20 pm in a city. What time is it in the 5th time zone to the East?

9. If it's **3:50 A.M** in Sydney, what time is it in the **11th time zone West**?

10. A car travels at a speed of **70 km/h**. Express this speed in **metres per second (m/s)**.

11. In the East African Safari Rally, Mansoor drove at a speed of 40m/sec. What speed was he driving in kilometres per hour?

Assessing attitudes and values

12. A delivery driver must reach a destination **100 km away in 2 hours**.
    a) Is it safe to speed if traffic is heavy?
    b) What should they do instead?
    c) Why is it important for drivers to **obey speed limits** on roads?

13. A friend says, "Time zones don't matter because we have phones that adjust automatically." Do you agree? Give reasons.

14. If two athletes A and B finish a race at the **same time** but the athlete A ran **faster**, is this possible? Who can win the price?

UNIT 9

SIMPLE INTEREST AND PROBLEMS INVOLVING SAVING

Key Unit Competence: You will be able to work out simple interest and solve problems involving saving.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- Define different terms such as simple interest, interest rates, principle and time.
- Explain the importance of saving.

Skills:

- Solve problems involving the calculation of simple interest.
- Solve problems involving savings.

Attitudes and values:

- Show confidence when calculating simple interest to be paid by the Saving and Credit Association (SCA).
- Appreciate the importance of simple interest in daily life situations.
- Appreciate the importance of saving in daily life.

9.0 Introduction

In this unit, you will explore how money grows over time through simple interest as the reward for saving or a cost for borrowing. You will learn how to calculate interest using the formula "Interest = (Principal × Rate × Time)" and apply it to real-life situations, such as saving in a bank or taking a small loan. You will also solve problems involving savings, helping you understand how setting aside money regularly can lead to bigger amounts in the future. By mastering these skills, you'll be able to make financial decisions and see how mathematics helps us plan for goals like buying books or starting a business.

Introductory Activity

Mukama put 200,000 Frw in the saving group for a year. The Saving group gave him 10 out of each 100 Frw for using his money.

a) How much more money was given to Mukama after 1 year?
b) Why do you think we should save our money in the ikibina Saving and Credit Association?
c) How does interest benefit us in our daily life?
d) Can you give different possibilities of using money and getting interest without saving it in the bank?

9.1 Calculating the simple interest

Activity 9.1

Visit a Saving and Credit Association (Ikimina) if available.

Ask how much money the SCA gives to the client who saves the money for example 100,000Frw in the Saving after 1 year.

Write down the information on a sheet of paper.

Back to class, make a class discussion

Summary

The money borrowed, saved or lent is principal (P).

The percentage used to calculate interest is Rate (R).

Rate is the amount on everyone hundred of the principal that is earned or paid back.

The period in years that the principal is invested is time (T).

The additional amount offered or paid is Simple Interest (S.I).

$$\text{Simple interest(S.I)} = P \times \frac{R}{100} \times T$$

When we take the decimal number $r = \frac{R}{100}$, we get:

$$\text{Simple interest(S.I)} = P \times r \times T$$

Example

Given; principal (P) = 100,000 Frw, interest rate (R) = 10% p.a and Time (T) = 2 years. Find the simple interest (S.I).

Solution

P = 100,000 Frw, R = 10%, T = 2 years, S.I = ? Frw

$$\text{Simple interest} = P \times \frac{R}{100} \times T = 100{,}000 \times \frac{10}{100} \times 2 = 10{,}000 \times 2 = 20{,}000 \text{ Frw.}$$

Therefore, the simple interest is 20,000 Frw.

Application activity 9.1

Calculate simple interest (S.I) given,

1. Principal = 400,000 Frw, Rate = 5% p.a, Time = 1 year.
2. Principal = 650,000 Frw, Rate = 12% p.a, Time = 3 years.
3. Principal = 800,000 Frw, Rate = 15% p.a, Time = 4 years.

9.2 Calculating simple interest after a given number of months

Activity 9.2

1. Express the following in terms of years:

   (a) 4 months (b) 6 months (c) 8months (d) 3months

Change the following into fractions:

2. a) $2\frac{1}{2}\%$ b) $12\frac{1}{2}\%$ c) $6\frac{1}{4}\%$ d) $37\frac{1}{2}\%$

What are your answers?

Summary

If rate is a fractional percentage, change it into a common fraction first.

If time is in months, change it into years first.

Remember 12 months make one year. Express the months given out of 12.

Example

Given; principal (P) = 1,680,000 Frw, Interest rate (R) = $12\frac{1}{2}\%$ p.a and Time (T) = 8 months. Find the simple interest.

Solution

P = 1,680,000 Frw, R = $12\frac{1}{2}\%$ T = 8 months,

Change 8 months to years = $\frac{8}{12}$ of a year. S.I = ? Frw

Change $12\frac{1}{2}\%$ into a fraction

$$\frac{12\times2+1}{2}\times\frac{1}{100}=\frac{25}{2}\times\frac{1}{100}=\frac{25}{200}$$

$$\text{Simple interest}=P\times\frac{R}{100}\times T=1{,}680{,}000\times\frac{25}{200}\times\frac{8}{12}=1400\times4\times25=140{,}000\text{frw}$$

Therefore, the simple interest is 140,000 Frw.

Application activity 9.2

Find the simple interest:

1. Principal = 1,200,000 Frw, interest rate = $2\frac{1}{2}\%$ p.a and Time = 6 months.
2. Principal = 10,050,000 Frw, interest rate = $6\frac{1}{5}\%$ p.a and Time = 2years.
3. Principal = 1,800,000 Frw, interest rate = $16\frac{1}{2}\%$ p.a and Time = 3 months

9.3 Solving problems involving simple interest

Activity 9.3

Phany deposited 6,500,000 Frw in their Saving and Credit Association (SCA).

The SCA promises to give her interest rate of 10% per year in 3 years.

What will be her simple interest after the 3 years?

Explain how you get it and then present your working out to the class.

Summary

Example 1

A man deposited 45,000 Frw in a Saving group that offers an interest rate of $3\frac{1}{2}\%$ per year (per annum). How much interest will he get in 2 years?

Solution

Principal (P) = 45,000 Frw, Rate (R)= $3\frac{1}{2}\%$, time (T) = 2 years, simple interest=?

$$\text{Simple interest} = \text{P} \times \frac{\text{R}}{100} \times \text{T}$$

$$= 45{,}000 \times \frac{7}{200} \times 2 = 450 \times 7 \times 1 = 3{,}150 Frw$$

Example 2

Ndahiro wanted to buy a motobike at 1,000,000 Frw.

He borrowed from the Saving group at a rate of 8% per annum. What interest did he pay after 6 months?

Solution

P = 1,000,000 Frw, R = 8%,

T = 6 months= $\frac{6}{12}$,

S.I = ?

$$\text{Simple interest} = \text{P} \times \frac{\text{R}}{100} \times \text{T}$$

$$= 1{,}000{,}000 \times \frac{8}{100} \times \frac{6}{12} = 10{,}000 \times 4 = 40{,}000 Frw.$$

Application activity 9.3

1. Karoli deposited 60,000 Frw in the Saving group. The interest rate was 10% per year. Calculate the simple interest after 5 months.
2. Mutuyimana borrowed 120,000 Frw from a Saving group at an interest rate of 20% per year. How much interest did she pay after 3 year?
3. Uwase saved 350,000Frw in a Saving group. How much interest did she earn in 6 months if the rate was 10% per year?
4. Mugenzi borrowed 150,000 Frw at an interest rate of 30% per year. Calculate the interest he earned after 4 years

9.4 Calculating interest rate

Activity 9.4

The formula for simple interest is: SI = $p \times \frac{R}{100} \times T$

- Multiply the formula by 100 both side.
- Divide both sides by P and simplify.
- Divide both sides by T. Simplify, then write the achieved formula.
- What do you notice. Explain your working out steps to the class.

Summary

Read and interpret the question correctly.
Identify the principal, simple interest and time.

To find rate, substitute for principal, simple interest and time

To find rate solve for R in $R = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$

Rate is a percentage

Example

Calculate the interest rate given Principal = 100,000 Frw, Time = 2 years, interest = 20,000 Frw.

Solution

P = 100,000 Frw, T = 2 years, S.I = 20,000 Frw, R = ?

$$R = \frac{\text{simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

$$R = \frac{20,000 \times 100}{100,000 \times 2} = \frac{10}{1} = 10\%$$

Application activity 9.4

Find the interest rate given:

1. Principal = 1,200,000 Frw, Time = 4 years and interest = 400,000 Frw.
2. Principal = 960,000 Frw, Time = 2 years and interest = 60,000 Frw.
3. Principal = 100,000 Frw, Time = 1 year and interest = 25,000 Frw.
4. Principal = 1,440,000 Frw, Time = $1\frac{1}{2}$ years and interest = 72,000 Frw

9.5 Solving problems involving interest rate

Activity 9.5

Your father has a loan of 800,000 Frw from a bank.

In one year, he pays interest of 96,000 Frw.

Help him to calculate the interest rate.

Summary

$$R = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

The percentage used to calculate interest is the rate.

Rate is worked out by substituting simple interest, principal and time.

Example 1

Uwacu deposited 8,500 Frw on her account. After 2 years, she earned an interest of 2,450 Frw. Find the interest rate she was offered.

Solution

P = 8,500 Frw, S.I = 2,450 Frw

T = 2 years, R = ?

$$R = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

$$R = \frac{2,450 \times 100}{8,500 \times 2} = \frac{245}{17} = 14\frac{7}{17}\% \text{ or } 14.4\%$$

Therefore, rate is 14.4%.

Example 2

Zaninka deposited 60,000 Frw on her savings account. After 8 months, the interest gained was 4,000 Frw. Calculate the rate of interest.

Solution

P = 60,000 Frw, S.I = 4,000 Frw

T = 8 months, R = ?

$$R = \frac{\text{Simple interest} \times 100}{\text{Principal} \times \text{Time}}$$

$$R = \frac{4,000 \times 100}{60,000 \times \frac{8}{12}} = \frac{4,000 \times 100 \times 12}{60,000 \times 8} = 5 \times 2 = 10\%$$

Therefore, rate is 10%.

Application activity 9.5

1. Akaliza banked 60,000 dollars and earned an interest of 15,000 dollars in 4 years. What was the rate of interest?
2. Mugenzi borrowed 320,000 Frw from bank for one year. He payed an interest of 32,000 Frw. Find the interest rate.
3. Uwacu banked 50,000 Frw in a bank and earned an interest of 15,000 Frw after 3 years. What was the rate of interest?

9.6 Calculating principal

Activity 9.6

On a slip of paper, write Simple Interest formula.

Multiply both sides by 100.

Divide both sides by R.

Then divide both sides by T.

Write the expression as an equation equated to P.

Now attempt this:

Write 20,000 Frw on a slip of paper. Multiply it by 100.

Divide the product by 10=R, then by 4=T.

What is your result? Can you explain how to find the principal P when the time , interest rate and simple interest are given?

Summary

Principal is the money saved, borrowed or lent.

$$\text{Principal} = \frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}}$$

Make sure the rate is given as a percentage not as a decimal number.

Example

Find the principal given simple interest = 10,000 Frw, interest rate = 5% and time = 4 years

Solution

S.I = 10,000 Frw, R = 5% p.a, T = 4 years, P = ? Frw

$$\text{Principal} = \frac{\text{Simple interest} \times 100}{\text{Rate} \times \text{Time}} = \frac{10,000 \times 100}{5 \times 4} = 10,000 \times 5 = 50,000$$

Therefore, the principal is 50,000 Frw.

Application activity 9.6

Find the principal given:

1. Simple interest is 2,000 Frw, rate is 2% p.a and time is 2 years.
2. Simple interest is 45,000 Frw, rate is 3% and time is 9 months.
3. Rate is 4% p.a, time is 3 years and simple interest is 60,000 Frw.
4. Time is 5 years, simple interest is 72,000 Frw and rate is 20% p.a.

9.7 Solving problems involving principal

Activity 9.7

Alpha deposited some money in the bank. If at the rate of 4% in 2 years he earned a simple interest of 5,000 Frw, what is the money he banked?

Explain how you get it.

Summary

Read and interpret the question correctly.

Principal is the money saved, borrowed or lent.

$$\text{principal} = \frac{\text{simple interest} \times 100}{\text{rate} \times \text{Time}}$$

Example 1

Kankera saved money in a bank that gives simple interest at rate of 8% per year. She earned interest of 480,000 Frw in 3 years. How much did she save?

Solution

Rate = 8%, Simple Interest = 480,000 Frw, Time = 3 years, Principal = ?

$$\text{Principal} = \frac{\text{simple interest} \times 100}{\text{rate} \times \text{Time}}$$

$$= \frac{480,000 \times 100}{8 \times 3} = 20,000 \times 100 = 2,000,000 Frw$$

Therefore, principal is 2,000,000 Frw.

Example 2

An amount of money gained an interest of 14,400 Frw. It was invested for 9 months. If the simple interest rate is 6%, calculate principal.

Solution

R = 6%, S.I = 14,400 Frw, T = 9 months

$$P = \frac{14,400 \times 100}{6 \times \frac{9}{12}} = \frac{14,400 \times 100 \times 12}{6 \times 9}$$

$$P = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{Time}}$$

$$= 1,600 \times 2 \times 100 = 320,000 Frw$$

Therefore, principle is 320,000 Frw

Application activity 9.7

1. Mary banked money at 5% simple interest rate and earned an interest of 5,000 Frw in 5 years. How much did she bank?
2. Rene borrowed money that earned her 45,600 Frw as interest in 4 years at a simple interest rate of 12%. Calculate the amount she borrowed.
3. Jeanne banked money in a bank that gives an interest rate of 10% per year. 4 years later, he earned an interest of 65,000 Frw. What was the principal?

9.8 Calculating the time

Activity 9.8

Get a slip of paper.

Take the formula for the simple interest and multiply by 100 both sides.

Now divide the product by the product of P and R both sides.

What expression is formed?

Now, explain how to find the time T when the simple interest, the rate and the principal are given.

Summary

Time is the period in years in which the principal is invested.

Time is calculated by substituting simple interest, principal and rate

$$\text{Time} = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{rate}}$$

Example

Find time given, the simple interest is 12,000 Frw, Principal is 144,000 Frw and rate is 20% per annum (p.a).

Solution

S.I = 12,000 Frw, P = 144,000 Frw, R= 20%, T = ?.

$$\text{Time} = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{rate}} = \frac{12,000 \times 100}{144,000 \times 20} = \frac{5}{12}$$

Therefore, time is $\frac{5}{12}$ of a year or 5months.

Application activity 9.8

Calculate time given:

1. Simple Interest is 20,000 Frw, principal is 200,000 Frw and interest rate is 5% per year.
2. Principal is 100,000 Frw, simple interest is 10,000 Frw and interest rate is 10% p.a.
3. Interest rate is 5% p.a, principal is 1,200,000 Frw and simple interest is 40,000 Frw.

9.9 Solving problems involving time

Activity 9.9

How long do you take to earn 4,000 Frw as simple interest at the rate of 4% p.a. when you deposit 50,000 Frw in a local bank?

Explain your working out to the class.

Summary

Time is always expressed in years.

$$\text{Time} = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{rate}}$$

Example 1

Eva deposited 15,000 Frw in a bank that offers a simple interest rate of 3% per year. If she got a simple interest of 1,800 Frw, how long had she banked the money?

Solution

Principle = 15,000 Frw, Rate = 3%,

Interest = 1,800 Frw, Time = ?

$$\text{Time} = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{rate}}$$

$$= \frac{1,800 \times 100}{15,000 \times 3} = \frac{60}{15} = 4$$

Therefore, the time was 4 years.

Example 2

Kamanzi borrowed 1,200,000 Frw from Bank. The bank offered 6% per year. He paid a simple interest of 240,000 Frw. How long did he use the money?

Solution

P = 12,000,000 Frw,

S.I = 240,000 Frw, T = ?

$$\text{Time} = \frac{\text{simple interest} \times 100}{\text{principal} \times \text{rate}}$$

$$= \frac{240,000 \times 100}{1,200,000 \times 6} = \frac{40}{12} = 3\frac{1}{3}$$

Therefore, the time was $3\frac{1}{3}$ years.

Application activity 9.9

1. Kayitesi deposited 18,000 Frw in a bank that gives an interest rate of 10% per year. How long did it take her to get 1,800 Frw as simple interest?
2. Mr Ntwari borrowed 50,000 Frw from a bank that gives an interest rate of 10% p.a. How long did he use the money if he earned a simple interest of 25,000 Frw?
3. Nsenga got asimple interest of 70,000 Frw at a rate of 20% per year. How long did it take him if he deposited 350,000 Frw?
4. Munyandekwe lent Mukandori 240,000 Frw at an interest rate of 5% per year. After how long will he get a simple interest of 30,000 Frw?

9.10 Calculating the amount of money

Activity 9.10

Take a slip of paper.

Write principal P under the interest rate R.

If Mugabe gets the simple interest SI after the time T. How much money does Mugabe have?

Explain your working out to your classmates.

Summary

After getting the simple interest, the amount of money obtained is the sum of principal and simple interest.

To find amount, first calculate interest, then add it to principle.

Total amount = SI + P

Example

Calculate the amount of money obtained given the principal of 1,200,000 Frw, interest rate $12\frac{1}{2}\%$ p.a and time is 6 months.

Solution

$$S.I = P \times \frac{R}{100} \times T \qquad S.I = \frac{1,200,000}{100} \times 12\frac{1}{2} \times \frac{6}{12}$$

$$S.I = 1,200,000 \times \frac{25}{200} \times \frac{6}{12} = 3,000 \times 25 = 75,000 Frw$$

Therefore, the simple interest is 75,000 Frw.

Amount = Principal + Interest = 1,200,000 + 75,000 Frw = 1,275,000 Frw

Therefore, the amount is 1,275,000Frw

Application activity 9.10

Find the amount given:

1. Principal is 1,000,000 Frw, rate is 20% p.a, time is 2 years.
2. Rate is 12% p.a, time is 4 years, principal is 1,400,000 Frw.
3. Time is 3 years, rate is 25% p.a, principal is 2,840,000 Frw.
4. Principal is 380,000 Frw, time is 1 year, rate is 15% p.a.

9.11 Solving problems involving amount of money

Activity 9.11

Suppose you are a manager of certain Bank that offers simple interest.

Role-play a situation of a person who came to apply for a loan in your Bank.

Show how the loan is processed and then identify what the manager does in order to help the bank and the client to achieve their business objectives.

Explain your point of view.

Summary

Amount is the total sum of money in the bank after the principle and the interest earned over a given period.

Amount = Principle + interest earned.

Example

A school deposited 1,800,000 Frw on a fixed account in a Bank. The money took 3 months at an interest rate of 20% per year. How much money was on the account after 3 months if the bank offers simple interest.

Solution

P = 1,800,000 Frw, T = 3 months or $\frac{3}{12}$ year, rate = 20% p.a, S.I = ?, A = ?

$$\text{S.I} = \text{P} \times \frac{R}{100} \times \text{T}$$

$$\text{S.I} = 1{,}800{,}000 \times \frac{20}{100} \times \frac{3}{12} = 15{,}000 \times 2 \times 3 = 90{,}000 Frw$$

Simple interest is 90,000 Frw.

Amount= Principal + Interest = 1,800,000 Frw + 90,000 Frw = 1,890,000 Frw

Therefore, the amount is 1,890,000 Frw.

Application activity 9.11

1. Uwacu borrowed 840,000 Frw from a bank for one year. The interest rate is 18% per year. Calculate the amount she paid back.
2. A trader deposited 2,400,000 Frw in a bank. It offers an interest rate of 15% per year. What amount was on her account 6 months later?
3. Kibuye Dairy Cooperative society lent 1,500,000 Frw to each of its members for 2 years. They paid an interest of 5% per year. How much did every member pay back?
4. Kato borrowed 2,000,000 Frw from Jasmine for 8 months. He paid an interest of 12% per year. How much did he pay altogether?

9.12 Saving money in the bank or putting it in investments

Activity 9.12

You are aware that successful people, institutions and business people save money in the banks or in investments.

Suppose you have 20,000 Frw, write down different ways of saving it.

Discuss and give an example of the following:

i) the advantage of saving money in banks.

ii) the advantages of putting money in investments.

Present your working out to the class.

Summary

Savings is the amount of money kept after spending some of it on basic needs.

Savings is calculated by subtracting expenditure from the money owned.

Savings = Income - Expenditure.

Banks or investments give additional money to the saver or investor.

It is because they use the saved money to earn more money.

Example

Suppose you have 10,000 Frw. Make a budget and show how you can save some of it.

Solution

Budget: one dozen of books for 3,000 Frw, one graph book for 500 Frw, one hand kerchief for 500 Frw, six pens 1,000 Frw, one Maths set 2,000 Frw.

Expenditure = (3,000 + 500 + 500 + 1,000 + 2,000) Frw = 7,000 Frw

Expenditure = 7,000 Frw

Saving = Owned money – Expenditure = (10,000 – 7,000) Frw =3,000 Frw

Application activity 9.12

1. Given 15,000 Frw, show how you can save from it.
2. A man invested 1,500,000 Frw for 2 years. He earned an interest of 10% p.a.
   How much money did he find on his account?
3. Amani bought shares of 50,000 Frw each. He invested 4,000,000 Frw in a company. Each share realized a monthly interest of 5,000 Frw.
   a) How much interest did he earn after 1 year?
   b) Calculate the amount he had on his account altogether.

9.13 Solving problems involving savings

Activity 9.13

John wants to get an interest of 3,000 Frw in one year. How much must he invest for that period at 8%?

Explain your working step.

Summary

Savings is the amount of money kept after spending some of it on basic needs

Savings = Income – Expenditure

Investment is the act of allocating resources, typically money, with the expectation of generating a profit or achieving some form of benefit in the future.

Example 1

Gabriel saved money in a bank. He was offered 1,500 Frw monthly as interest. If he deposited 1,350,000 Frw. How much did he have on his account after 2 years?

Solution

1 year = 12 months

Time = 2 × 12 = 24 months

In 1 month, he got interest of 1,500 Frw

In 24 months he got interest of (1,500 × 12) = 180,000 Frw

Amount = Principal + Interest = 1,350,000 + 180,000 = 1,530,000 Frw.

Gabriel had 1,530,000 Frw after two years.

Example 2

Kayitesi has been paid 4,000 Frw by one of her business partners. She wants to save this money.

a) Advise her on the different methods of saving.
b) Portray the importance of saving to Kayitesi.

Solution

a) - She can deposit the money in a bank to earn interest.
   - She can invest the money by buying shares in different investments such as companies, co-operative societies, and others.
   - If it is the only amount she has, she should spend some of it on needs, but not all. She must save some in the bank or SACCO.

b) - Kayitesi earns more money in form of interest.
   She will improve her standard of living by earning more money in future.
   - She will always be having money available to sustain her needs.

Application activity 9.13

1. Gasana invested 3,500 US dollars in a trading company offering an interest of 5 US dollars weekly.
   a) How much interest will he have in 3 years?
   b) What will be the account balance?
2. Joseph deposited 100,000frw on a fixed saving account that earns 3% simple interest monthly.
   a) Find the interest earned after 5 years.
   b) Calculate the total amount on his account then.
3. Dusabe invested 2,000,000 Frw in a company. He bought shares. He was offered 10,000 Frw as interest every month. How much will he earn in 5 months?
4. 2,800,000 Frw is invested in a roofings company by a construction company monthly. An interest of 45,000 Frw is gained monthly.
   a) How much will theconstruction company realize as interest in 5months?
   b) Find the total amount on its account

9.14 End of unit assessment

Assessing knowledge and understanding

1. Define **simple interest** and write its formula.
2. If the principal is **100,000 Frw**, the rate is **5% per year**, and the time is **2 years**, calculate the simple interest.
3. What does the term **"principal"** mean in simple interest calculations?

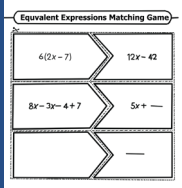
Assessing skills

4. Find simple interest earned on 150,000 Frw at an interest rate of 30% per year.
5. Mushumba earned an interest of 4,500 Frw for 2 year at 5% interest rate per year. What was the principal?
6. How long will 700,000 Frw take to gain an interest of 140,000 Frw at an interest rate of 5%?
7. Kamanzi deposited 1,280,000 Frw in a bank for 8 months. He realised an interest of 64,000 Frw. Calculate the interest rate the bank offered him.

Assessing attitudes and values

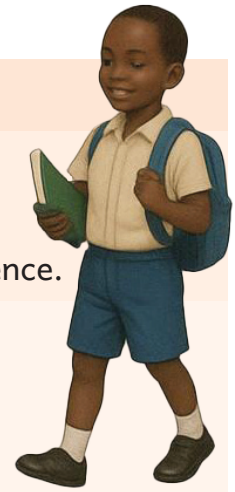
8. Why is it important to **save money in a bank instead of keeping it at home? Give two reasons**.
9. A friend says, "Taking a loan with high interest is okay because I'll pay it back later." Do you agree? Explain.
10. **If you have 25,000 Frw, show how you would spend it on the following:** school materials, clothes, betting, buy play materials and saving.

UNIT 10



EQUIVALENT EXPRESSIONS AND NUMBER SEQUENCES

Key Unit Competence: You will be able to write sequences of whole numbers, fractions and decimals.



Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- Give examples of algebraic expressions and equivalent expressions.
- Explain how to find the rule for determining the next term in a sequence.

Skills:

- Match equivalent expressions.
- Determine the rule for a sequence.
- Find the missing number in a sequence following a particular rule.
- Solve real life problems involving a sequence with a determined rule.

Attitudes and values:

- Appreciate the importance of orderliness in finding out different terms of sequence and extend it to real life situations.
- Show the faithfulness to the group members when solving a problem involving sequences.

10.0 Introduction

In this unit, you will explore how different Mathematics expressions can have the same value and discover the rule used in different number sequences. You will learn to simplify expressions, compare them, and identify rules that govern sequences. These skills will help you solve problems faster and see how math works in an organized way.

Equivalent expressions help us calculate things like sale prices or shared costs, while number sequences appear in music beats, stair steps, and even nature. By understanding these concepts, you'll be able to spot patterns in daily life, manage money better, and think logically.

Introductory Activity

Pour 3 litres of water in a small jerrycan and mark the point. Remove the water and then get a soda bottle of 300 ml. Put the water in a bottle at a time as you pour in the same jerrycan until you fill it to the same level. What do you observe?

a) Find out the number of soda bottles to fill the jerrycan at the same level as for 3 litre bottles.
b) After your experience, are 3 litres of water equivalent to 10 times the water in the bottle of soda containing 300 ml? Justify your answer.

10.1 Algebraic expressions

Activity 10.1

Imagine a letter *m* or *x* that stands for a number you don't know. Write the following algebraic expressions:

a) Twice a number added to 4.
b) The product of 2 and a number minus 3.
c) The difference between a number and 5.
d) A number multiplied by 3 take away 1.

Summary

When writing an algebraic expression from words,

1. **Select** a letter (like x, n) to stand for the unknown.
   Example: "A number plus 3" → **n + 3**
2. **Identify Key Words:**

   Addition (+): sum, total, more than, added to, increased by .
   Example: "5 more than x" → **x + 5**

   Subtraction (−): difference, less than, decreased by, minus, fewer than. Example: "8 less than y" → **y− 8** (Order matters!)

   Multiplication (×): times, product, multiplied by, of, twice (×2), double (×2). Example: "Twice a number" → **2n**

   Division (÷): divided by, quotient, per, half (÷2).
   Example: "k divided by 3" → **k ÷ 3** or $\frac{K}{3}$
3. **Watch the Order**
   Example: "10 less than a number" → **n − 10** (Not 10 − n)
   Example: "5 subtracted from a number" → **x − 5**

Grouping with Parentheses: Use ( ) when operations need to happen first. Example: "7 times the sum of 2 and x" → **7(2 + x).**

Example

1. Write this algebraic expression. Add 7 to m, then subtract 3.

Solution

Add 7 to m then subtract 3: (m + 7) – 3

2. Write this algebraic expression. Twice x add thrice y.

Solution

Twice x add thrice y: 2x + 3y.

3. Write this algebraic expression: The difference between a and b divided by 4.

Solution

The difference between a and b divided by 4: $\frac{a-b}{4}$

4. Write this algebraic expression. The product of 2 and y added to the product of 3 and z.

Solution

It becomes 2y + 3z

5. The meaning of the following algebraic expressions:
   a) 2x+1 : the sum of twice a number and 1.
   b) 3m – 6 : Subtract 6 from the three times of a number.

Application activity 10.1

Write the following algebraic expressions for these:

1. 3 times the difference between p and q.
2. Divide thrice the product of m and n by 7.
3. Divide the sum of y and 6 by 5.
4. Twice x, take away 6, then multiply the result by 3.

10.2 Equivalent expressions

Activity 10.2

Collect 6 pens and 4 rubbers.

Form 2 groups of each item.

Write an algebraic expression relating the collected items to the grouped ones.

Summary

Equivalent expressions are different mathematics expressions that always give the same answer when we consider every possible value of the unknown.

Expression on the left side of the sign equal must be equal to the expression on the right side.

To find equivalent expressions, follow these simple rules:

1. Do the same math to both sides

If you add, subtract, multiply, or divide one part of an expression, you must do the same to the other side to keep it equal.

Example: $3x = 2$ is equivalent to $3x+1 = 2+1$ (add 1 to both sides).

2. Combine like terms

Group terms with the same variable or constants:

Example: $2x+5+x-3 = (2x+x) + (5-3) = 3x+2$

3. Break down multiplication over addition/subtraction:

Example: $2(x+4) = 2x+8$.

4. **Make factors from addition or subtraction**: Example: $4x+6 = 2(2x+3)$.

5. **Divide numerator and denominator by the same number**:

Example: $\frac{6x}{8} = \frac{3x}{4}$ (divided by 2).

The expression on the left has to be equivalent to the expression on the right.

Examples

1) Verify if $(3a + 6) = 3(a + 2)$

Solution

$(3a + 6) = 3(a + 2)$?

Open brackets for the right side.

$3a + 6 = (3 \times a) + (3 \times 2)$?

$3a + 6 = 3a + 6$

Therefore, $3a + 6$ is equivalent to $3(a + 2)$.

2) verify if $(y - 4) = 2(y+2)$

Solution

$y - 4 = 2(y+2)$?

$y - 4 = (2y) + (2 \times 2)$?

$y - 4 \neq 2y + 4$?

It is not correct. $y - 4$ is not equal to $2y + 4$.

Application activity 10.2

Verify and explain if the following expressions are equal

a) $4a + 2b = 2(2a + b)$

b) $5(x - 5) = 5x - 5$

c) $3(3y + 4) = 9y + 12$

d) $6q - 4 = 2(3q - 2)$

10.3 Finding the missing consecutive numbers of a sequence

Activity 10.3

- Collect 100 straws.
- Distribute them to 5 learners in the order 1st to 5th.
- Give the 1st two straws. Five straws to the 2nd and 8 straws to the 3rd learner.
- Write the order of the given out straws on paper slips.
- How many straws should be given to the 4th and 5th learners?
- How do you find the number of straws to be given to the next learners?

  What is the order of increase?

Summary

In the activity 10.3, the number of straws to be given to learners make a sequence for which terms are increasing. The same number to be added to get the next term is the common difference.

To complete the missing number, find the **common differenc** .

Then, continue adding the common difference to find the next number and other missing numbers.

Example

Find the missing numbers. 3, 7, 11, _____, _____, _____

Solution

First find the order of increase.

This is done by finding the common difference.

So, $(7 - 3) = 4$, $(11 - 7) = 4$

The order of increase is adding 4 to the number to get the next number.

The 4th number is $(11 + 4) = 15$

5th number is; $(15 + 4) = 19$

6th number is $(19 + 4) = 23$

7th number is $(23 + 4) = 27$

So, the linear sequence is:

3; 7; 11; 15; 19; 23; 27; ...

Application activity 10.3

Find the missing numbers in the sequences below;

1. 3, 8, 13, 18, _____, _____, _____
2. 1, 4, 7, 10, _____, _____, _____
3. 2, 5, 8, 11, _____, _____, _____

10.4. Finding the missing consecutive fractions or decimals

Activity 10.4

- Write a number on a piece of paper.
- Add $\frac{1}{3}$ to the selected number and find the sum.
- Add $\frac{1}{3}$ to the previous fraction and write the sum.
- Continue adding $\frac{1}{3}$ to find other six consecutive fractions.
- Form a linear sequence for the fractions worked out.
- Explain your working out to the class.

Summary

To get the next fraction or decimal in a linear sequence, **find out the common difference** Then continue adding the common difference to find the missing fraction or decimal

Example

1. Find the missing decimal: 0.2, 0.5, 0.8, 1.1, _____,_____,_____

Solution

First find the order of increase.

(0.5 - 0.2 = 0.3), (0.8 - 0.5 = 0.3), (1.1 - 0.8 = 0.3)

The order of increase is adding 0.3. The 5^{th} decimal = (1.1 + 0.3) = 1.4

The 6^{th} decimal = (1.4 + 0.3) = 1.7; The 7^{th} decimal = (1.7 + 0.3) = 2.0;

The 8^{th} decimal = (2.0 + 0.3) = 2.3

Therefore, the linear sequence is: 0.2, 0.5, 0.8, 1.1, 1.4, 1.7, 2.0, 2.3; ...

2. Find the missing fraction 1, $1\frac{1}{2}, 2, 2\frac{1}{2}$_____, ____, _____

Solution

$\left(1\frac{1}{2}-1=\frac{1}{2}\right)$, $\left(2-1\frac{1}{2}=\frac{1}{2}\right)$, $\left(2\frac{1}{2}-2=\frac{1}{2}\right)$. The order of increase is adding $\frac{1}{2}$ to the fraction to get the next fraction.

The 4^{th} fraction = $\left(2\frac{1}{2}+\frac{1}{2}=\frac{6}{2}=3\right)$

The 6^{th} fraction $=3\frac{1}{2}+\frac{1}{2}=\frac{8}{2}=4$

The 5^{th} fraction $=3+\frac{1}{2}=\frac{7}{2}=3\frac{1}{2}$

The 7^{th} fraction $=4+\frac{1}{2}=\frac{9}{2}=4\frac{1}{2}$

Therefore, the linear sequence is $1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 4\frac{1}{2}$; ...

Application activity 10.4

Find the missing fraction or decimal in the sequences below:

1) $1; 2\frac{1}{2}; 4; 5\frac{1}{2}; \_\_; \_\_$,

2) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ ____,____,_____

3) 0.10, 0.13, 0.16, 0.19, ____._____,________

4) 2.03, 2.08, 2.13, 2.18,_____,_____,_____

10.5 Finding the general term or rule of a linear sequence

Activity 10.5

Write the linear sequence; 2, 5, 8, 11, _____,_____,_____

What is the order of increase, that is term to term?

Order the terms as 1^{st} , 2^{nd} , 3^{rd} , 4^{th} , etc.

Match each position by the increasing term.

What expression that can help you to find the number of each position referring to 2 as the first number?

Make a presentation of your working out in class.

Summary

To find the general rule for a sequence,

- Verify if the sequence is increasing by adding; then find the common difference **d**.
- Use multiply the positive (n) by the common difference, then add or subtract a constant.

Example

Find the general rule for the linear sequence below: 7, 13, 19, 25, 31, _____,_____,_____

Solution

First get the common difference. (13 – 7 = 6), (19 – 13 = 6), (25 – 19 = 6), (31 – 25 = 6)

Let the next term be n

Order of term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	n^{th}
Term	7	13	19	25	31	n
	(1 × 6) + 1	(2 ×6) + 1	(3 × 6) + 1	(4 × 6) + 1	(5 × 6) + 1	(n ×6) +1

(1 × 6) + ? = 7, (2 × 6) + ? = 13, (3 ×6) + ? = 19, (4 × 6) + ? = 25, (5 × 6) + ?= 31, (n × 6) + ?

The number added in the order is 1. So, (n × 6) + 1 = 6n + 1.

Therefore, the general rule is 1+ 6n.

The sequence is 7, 13, 19, 25, 31, **..., (1+ 6n),** ...

Application activity 10.5

Find the general term for the linear sequences below:

a) 5, 9, 13, 17_____,_____,_____

b) 2, 4, 6, 8, _____,_____,_____

c) 3, 11, 19, 27, 35, _____,_____,____,____

10.6 Finding the general term/rule of linear sequence for fractions and decimals

Activity 10.6

Write 1, $1\frac{1}{2}$,2, $2\frac{1}{2}$, 3, _____,_____,_____ on slips of paper

Work out the order of increase. Find the missing terms.

Applying the concept of finding the general term, find the same for the linear sequence you listed.

Summary

To find the general term,

First find the common difference, then multiply it by the order of term. If the product is not equal to the term, add or subtract a constant.

Example

1. Find the general rule for the linear sequence below $\frac{3}{4}, 1\frac{1}{2}, 2\frac{1}{4}, 3$ _____, _____, _____, _____

   Show all the necessary working steps.

Solution

The common difference is $\frac{3}{4}$ while the first number added is 0.

Order of term	1^{st}	2^{nd}	3^{rd}	4^{th}	n^{th}
Term	$\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{1}{4}$	**3**	**n**
	$1\times\frac{3}{4}$	$2\times\frac{3}{4}$	$3\times\frac{3}{4}$	$4\times\frac{3}{4}$	$n\times\frac{3}{4}$

Multiply the order of term by the common difference. Then find the constant to be added or subtracted.

$1\times\frac{3}{4}=\frac{3}{4}$, $2\times\frac{3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$, $3\times\frac{3}{4}=\frac{9}{4}=2\frac{1}{4}$, $n\times\frac{3}{4}=\frac{3}{4}n$

Therefore, the general rule is $\frac{3}{4}n$. Values of n are 1, 2, 3, etc

The sequence is $\frac{3}{4},1\frac{1}{2},2\frac{1}{4},3$, ..., $\frac{3}{4}n$, ...

2. Find the general rule for the linear sequence below: 0.1, 0.4, 0.7, 1.0_____,_____,_____.

Solution

The common difference is 0.3.

Order of term	1^{st}	2^{nd}	4^{th}	n^{th}
Term	0.1	0.4	1.0	?
	1 × 0.3 - ?	2 × 0.3 - ?	4 × 0.3 - ?	n × 0.3 - ?
	(1 × 0.3) - 0.2 = 0.1	(2 × 0.3) - 0.2 = 0.4	(4 × 0.3) - 1.0 = 1.0	(4 × 0.3) - 0.2 = 0.3n - 0.2

Multiply the order of term by the common difference. Then find the constant to be subtracted,

The common difference is 0.3, the constant to be subtracted is 0.2.

Therefore, the general rule is 0.3n - 0.2.

The sequence is 0.4; 0.7; 1.0; ...**(0.3n - 0.2);** ...

Application activity 10.6

Find the general term/rule for the linear sequences below:

a) $1\frac{1}{5};1\frac{2}{5};1\frac{3}{5};1\frac{4}{5}$____,_____,_____

b) $2\frac{2}{3};3\frac{1}{3};4;4\frac{2}{3}$____,_____,_____

c) $3\frac{1}{4};3\frac{1}{2};3\frac{3}{4};4$ ____,_____,_____

10.7 Finding the missing number or n^{th} term in a linear sequence

Activity 10.7

Find the general rule for the linear sequence: 1, 3, 5, 7____, _____, _____

Now, taking n to be the order of term, calculate to find the 12^{th} term.

Explain to the class.

Summary

To find the n^{th} term in a given sequence,

- first find the general rule.
- Multiply the position by the common difference and decide on how to get each term including the n^{th} term.
- Use the general rule to find the n^{th} term, by substitution.

Example

Find the 20^{th} term in the sequence: 2, 4, 6, 8, 10, 12____, _____, _____

Solution:

Find the common difference:

(4 – 2) = 2; (6 – 4) = 2; (10 – 8) = 2; (12 – 10) = 2.

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	n^{th}
2	4	6	8	10	12	?

Find the general rule:

(1 x 2) = 2; (2 x 2) = 4; (3 x 2) = 6; (4 x 2) = 8; (5 x 2) = 10; (6 x 2) = 12; (n x 2) = 2n

The general term is 2n, n is the term. The 20^{th} term is; 2 x 20 = 40.

The 20^{th} term is 40.

Application activity 10.7

Find the missing number

a) Find the 15th term in the sequence: 5, 8, 11, 14, 17, ____, _____, _____

b) Find the 19th term in the sequence: 1, 3, 5, 7, 9, ____, _____, _____

c) Find the 20th term in the sequence: 2, 7, 12, 17, 22, ____, _____, _____

d) What is the 30th term in the sequence: 3, 7, 11, 15, 19____, _____, _

10.8 Finding the missing fractions or n^{th} term in a linear sequence

Activity 10.8

Write the sequence: $\frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{3}{4}$ _____, _____, _____

Find the common difference. Write out the general rule for the fractional sequence.

Try to find the 10^{th} and 15^{th} terms using the general rule, but not by listing.

What do you observe? Explain your procedure to the class.

Summary

Find the general rule for the linear sequence.

Substitute the value of n^{th} term for n, then solve the equation to get the n^{th} term.

Example

Find the 12^{th} term in the sequence: $2\frac{3}{5}, 3\frac{1}{5}, 3\frac{5}{5}, 4\frac{2}{5}$ _____, _____, _____

Solution

Find the order of the increase $\left(3\frac{1}{5} - 2\frac{3}{5} = \frac{16}{5} - \frac{13}{5} = \frac{3}{5}\right)$,

$\left(3\frac{4}{5} - 3\frac{1}{5} = \frac{19}{5} - \frac{16}{5} = \frac{3}{5}\right)$; $4\frac{2}{5} - 3\frac{4}{5} = \frac{22}{5} - \frac{19}{5} = \frac{3}{5}$

Order the terms

1^{st}	2^{nd}	3^{rd}	4^{th}	n^{th}
$2\frac{3}{5}$	$3\frac{1}{5}$	$3\frac{5}{5}$	$4\frac{2}{5}$	?

Find the general rule

$\left(1\times\frac{3}{5} + ? = 2\frac{3}{5}\right), \left(2\times\frac{3}{5} + ? = 3\frac{1}{5}\right), \left(3\times\frac{3}{5} + ? = 3\frac{4}{5}\right), \left(4\times\frac{3}{5} + ? = 4\frac{2}{3}\right), \left(n\times\frac{3}{5} + ?\right)$

The constant added is 2.

The general rule is: $\left(n\times\frac{3}{5} + 2 = \frac{3}{5}n + 2\right)$ values of n are 1, 2, 3, 4, etc

The 12^{th} term$=\left(\frac{3}{5}n + 2\right) = \left(\frac{3}{5}\times 12 + 2\right) = \frac{36}{5} + 2 = \frac{36+10}{5} = \frac{46}{5} = 9\frac{1}{5}$.

Application activity 10.8

Find the missing term below:

1. Find the 7^{th} term in the sequence: $\frac{5}{6}, 1\frac{1}{3}, 1\frac{5}{6}, 2\frac{1}{3}$ _____, _____, _____
2. Find the 11^{th} term in the sequence: $\frac{2}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}$ _____, _____, _____
3. Find the 14^{th} term in the sequence: $1\frac{1}{3}, 2, 2\frac{2}{3}, 2\frac{1}{3}$ _____, _____, _____

10.9 Finding the number sequence using the general term or rule

Activity 10.9

Write 4n - 1 on slips of paper. Substitute the order 1^{st}, 2^{nd}, 3^{rd}, 4^{th} and 5^{th} for n to complete the table.

Order of term	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
Solving for 4n - 1	_____	_____	_____	_____	_____
	_____	_____	_____	_____	_____

Write the linear sequence worked out. Make presentation to the class.

Summary

To find the linear sequence, substitute the order of term for n in the general term/rule. Then solve.

Example

1. Given 6n + 1, find the linear sequence.

Solution

Write the order of terms: 1^{st}, 2^{nd}, 3^{rd}, 4^{th}. Substitute the order of term for n, then solve

1^{st} order = 6n + 1 = 6 x n + 1 = 6 x 1 + 1 = 6 + 1 = 7

2^{nd} order = 6n + 1 = 6 x n + 1 = 6 x 2 + 1 = 12 + 1 = 13

3^{rd} order = 6n + 1 = 6 x n + 1 = 6 x 3 + 1 = 18 + 1 = 19

4^{th} order = 6n + 1 = 6 x n + 1 = 6 x 4 + 1 = 24 + 1 = 25

Therefore, the linear sequence is: 7, 13, 19, 25,_____, _____, _____

2. Given $\frac{1}{2}n + 8$, find the linear sequence

Solution

Write the order of terms: 1^{st}, 2^{nd}, 3^{rd}, 4^{th}. Substitute the order of term for n, then solve.

1^{st} order = $\frac{1}{2}n+8=\frac{1}{2}\times1+8=\frac{1+16}{2}=\frac{17}{2}=8\frac{1}{2}$

2^{nd} order = $\frac{1}{2}n+8=\frac{1}{2}\times2+8=$ =1+8=9

3^{rd} order = $\frac{1}{2}n+8=\frac{1}{2}3+8=\frac{3}{2}+8=\frac{3+16}{2}=\frac{19}{2}=9\frac{1}{2}$

4^{th} order = $\frac{1}{2}n+8=\frac{1}{2}\times4+8=\frac{4}{2}+8$ =2+8=10

Therefore, the linear sequence is: $8\frac{1}{2}$, 9, $9\frac{1}{2}$, 10.

Application activity 10.9

Find the number sequences for the general rules below

a) 2n b)2n + 3 c) 3 n - 1 d) $\frac{3}{4}n$

10.10. End unit assessment

Assessing knowledge and understanding

1. Write algebraic expressions for:
   a) Subtract 6 from n, then multiply by 2.
   b) The sum of m and n divided by 4
2. Define: a) What is a linear sequence?
   b) What does "consecutive terms" mean in a sequence?
3. Identify: a) The common difference in: 3, 7, 11, 15, ---,---,
   b) The next term in: 1, 4, 7, 10, ---, ---,

Assessing skills:

4. Find the equivalent expressions.
   a) 3(2x – 4) = 2(3x – 6)
   b) $\frac{1}{2}$(4y + 2) = 2y + 1
   c) 14(2k – 1) = 7(4k – 2)

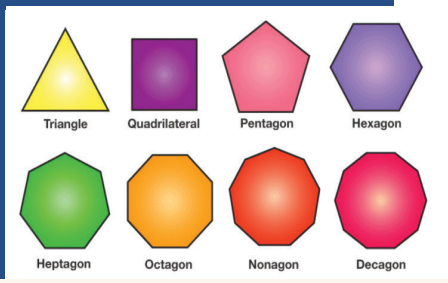
5. Find the missing consecutive numbers.
   a) 1, 5, 9, 13, _____, _____, _____
   b) 3, 7, 11, 15, _____, _____, _____
6. Find the general rule for the linear sequences below:
   (a) 1, 4, 7, 10, 13, _____, _____, _____
   (b) 2, 6, 10, 14, _____, _____, _____
7. (a) Find the 8th term in the sequence; 3, 7, 11, 15, _____, _____, _____
   (b) What is the 39th term in the sequence; 1, 6, 11, 16, _____, _____, _____
8. Find the missing fractions in the sequences below

   $\frac{2}{3}; 1\frac{1}{3}; 2; 2\frac{2}{3}; 3\frac{1}{3}; \_\_; \_\_; \_\_$

Assessing attitudes and values:

9. Explain why understanding sequences is important in real life (give two examples).
10. A student says "All number patterns follow simple rules." Do you agree? Explain your reasoning.
11. In a savings plan, you save 5Frw the first week, 10 Frw the second week, 15 Frw the third week, etc.:
    a) What's the pattern?
    b) How is this similar to the sequences we studied?
    c) Why is recognizing such patterns useful?

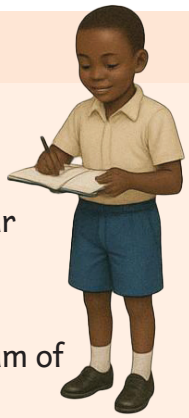
UNIT 11



REGULAR POLYGONS AND BEARINGS

Key unit competence: You will be able to use triangle, angle properties and compass directions to solve real-world problems involving regular polygons and bearings.

Learning objectives: By the end of this unit, you should be able to:



Knowledge and understanding:

- Define a regular polygon.
- Give the formulae used to calculate the perimeter and area of a regular polygon.
- Explain directions and bearing using compass points.
- Understand and use the angle sum of a triangle to determine the angle sum of a polygon.

Skills:

- Derive the interior angle of a regular polygon.
- Find the sum of interior/exterior angles of a regular polygon using the angle sum of a triangle.
- Calculate the length of the side, apothem, perimeter and areas of regular polygons.
- Use bearings to define direction.

Attitudes and values:

- Work systematically when investigating mathematical challenges involving regular polygons.
- Appreciate the importance of regular polygons in everyday life activities.
- Appreciate the relevance of bearings in daily life.

10.0 Introduction

Regular polygons are two dimensional shapes with all sides and angles equal. Common examples include equilateral triangles, squares, and regular pentagons, which appear frequently in house construction, engineering, and nature. Understanding their properties such as side length, internal angles, and symmetry helps in designing good shapes, creating patterns, and solving spatial problems. Bearings, on the other hand, are a system used in navigation to describe direction, measured in degrees from a fixed reference point, usually north. They provide a precise way to indicate movement or position, essential in fields like aviation, maritime travel and surveying.

Introductory Activity

The concept of polygon is used for making different objects. Observe the classroom and identify objects with the structure polygons and try to name them basing on the number of sides and angles.

a) Are there in your class objects of 3 sides and 3 angles?

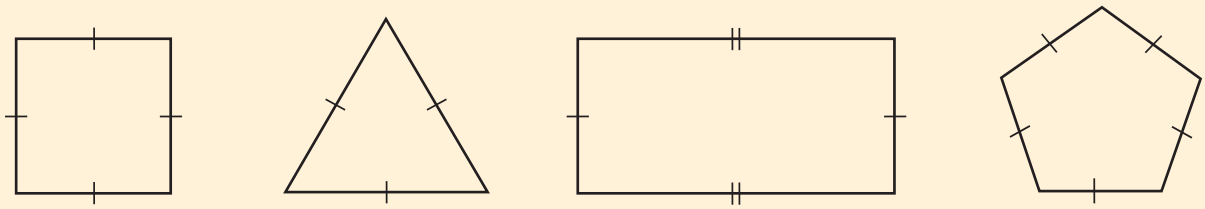
b) Are there any objects of 4 sides and 4 angles?

c) Select those with equal sides and equal angles. Then, name them as regular polygons.

11.1 Definition of Polygon and their Example

Activity 11.1

Name the shapes below.



Observe the above shapes then define them.

Which ones have equal sides and equal angles?

Explain the differences and similarities between the polygons.

Summary

A regular polygon has equal sides and equal angles, otherwise kite is irregular.

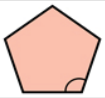
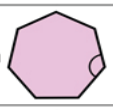
We name a polygon basing on its number of sides.

Example of regular polygons: equilateral triangle, square, pentagon, hexagon, heptagon, octagon, nonagon, decagon.

Example

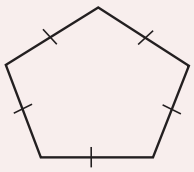
A polygon is any 2-dimensional shape formed with connected straight line segments called sides. The table shows the names of regular polygons and their corresponding number of sides.

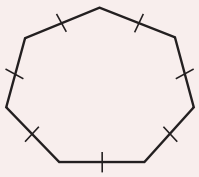
Example of regular polygon:

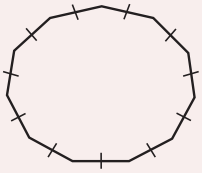
Name	Sum of Interior Angles	Each Interior Angle
Equilateral Triangle 	180°	60°
Square 	360°	90°
Pentagon 	540°	108°
Hexagon 	720°	120°
Heptagon 	900°	128.57°
Octagon 	1080°	135°
Nonagon 	1260°	140°
Decagon 	1440°	144°

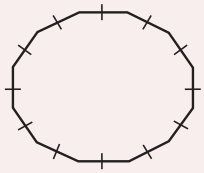
Application activity 11.1

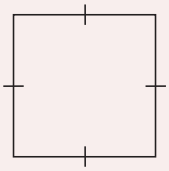
1. Name the following regular polygons.

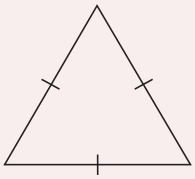
(a) 

(b) 

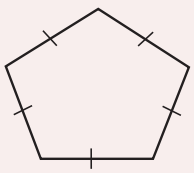
(c) 

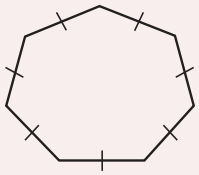
(d) 

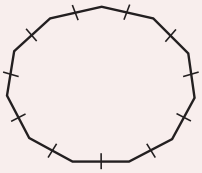
(e) 

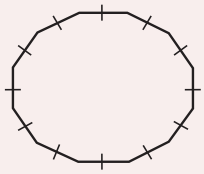
(f) 

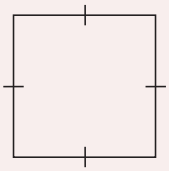
2. State the number of sides of the following polygons:

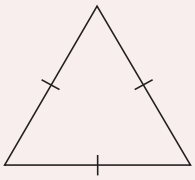
(a) 

(b) 

(c) 

(d) 

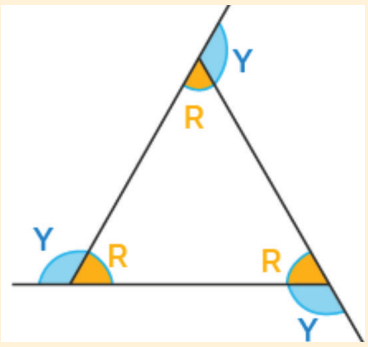
(e) 

(f) 

11.2 Investigating the interior and exterior angles of a polygon

Activity 11.2

- Draw the equilateral triangle on a sheet of paper.
- Extend the edges from the vertices with straight lines using a ruler.
- Measure the inside angle and its adjacent outside angles using a protractor. What names do you give the inside angle and the outside angle?
- Find the sum of the two angles you measured?
- Now draw a rectangle, pentagon, Octagon and a hexagon.
- Carry out the same steps you carried on a triangle.



What is your observation?

Summary

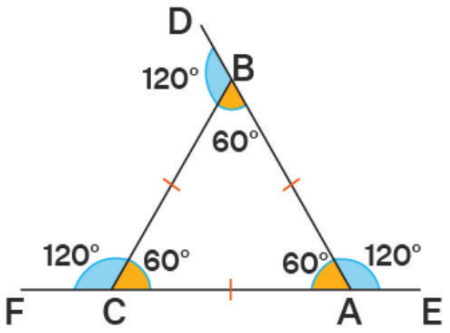
The inside angle of a polygon is called interior angle.

The outside angle of a polygon is called exterior angle.

The interior angle and exterior angles of a polygon are adjacent to each other.

The interior and exterior angles of a polygon are supplementary angles

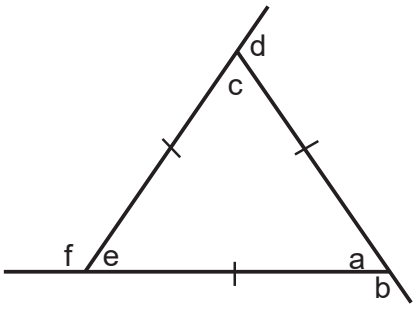
Interior angle + exterior angle = 180 degrees.



Example 1

Draw an equilateral triangle. Measure the interior and exterior angles using a protractor.

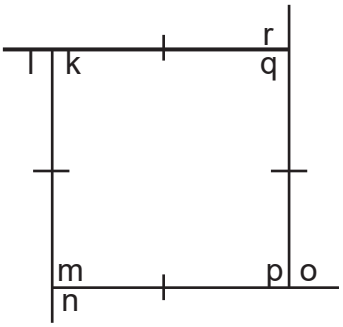
Solution



Example 2

Draw a square using a protractor. Measure the interior and exterior angles.

Solution



$\angle a = 60^0, \angle b = 120^0$

$\angle a + \angle b = 60 + 120^o = 180^o$

$\angle c = 60^o, \angle d = 120^o$

$\angle c + \angle d = 60o + 120^o = 180^o$

$\angle e = 60^o, \angle f = 120^o$

$\angle e + \angle f = 60^o + 120^o = 180^o$

∠a and ∠b supplement each other.

∠c and ∠d supplement each other.

∠e and ∠f supplement each other.

Therefore, the interior and its supplement exterior angles of a triangle add up to 180°.

$\angle q = 90^o, \angle r = 90^o$

$\angle q + \angle r = 90^o + 90^o = 180^o$

$\angle p = 90^o, \angle o = 90^o$

$\angle p + \angle o = 90^o + 90^o = 180^o$

$\angle k = 90^o, \angle l = 90^o$

$\angle k + \angle l = 90^o + 90^o = 180^o$

$\angle m = 90^o, \angle n = 90^o$

$\angle m + \angle n = 90^o + 90^o = 180^o$

∠q and ∠r supplement each other.

∠p and ∠o supplement each other.

∠k and ∠l supplement each other.

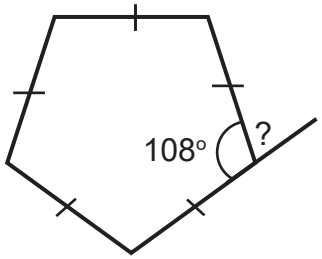
∠m and ∠n supplement each other.

Therefore, the interior and its supplement exterior angle of a square add up to 180°.

Example 3

The interior angle of a regular pentagon is 108degrees. Find its exterior angle.

Solution



Interior angle + exterior angle = 180 degrees. 108 degrees+ exterior angle = 180 degrees

Subtract 108 degrees from both sides,

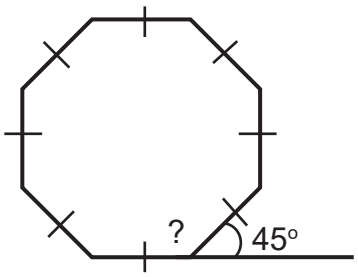
exterior angle = 180 - 108, Exterior angle = 72 degrees.

Therefore, the exterior angle is 72 degrees.

Example 4

The exterior angle of a regular octagon is 45 degrees. Find its interior angle.

Solution



Interior angle + exterior angle = 180 degrees
Interior angle + 45 degrees = 180 degrees

Subtract 45 degrees from both sides

Interior angle + 45 degrees - 45 degrees

= 180 degrees - 45 degrees

Interior angle = 135 degrees.

Therefore, the exterior angle is 135 degrees.

Regular polygon with 10 sides: Regular Decagon, 11 sides: Regular Hendecagon (or undecagon), 12 sides: Regular Dodecagon, 13 sides: Regular Tridecagon, 14 sides: Regular Tetradecagon, 15 sides: Regular Pentadecagon, 16 sides: Regular Hexadecagon, 17 sides: Regular Heptadecagon, 18 sides: Regular Octadecagon, 19 side: Regular Enneadecagon, 20 sides: Regular Icosagon.

Application activity 11.2

Use the polygon cards and sketches. Draw and measure to find the interior and exterior angles of these regular polygons (n ≤ 10).

(a)	Pentagon	(b)	Septagon	(c)	Nonagon
(d)	Hexagon			(f)	Heptagon
(g)	Decagon	(h)	Rectangle	(i)	Equilateral triangle
(j)	Square			(l)	Octagon

11.3 Investigating the sum of interior and exterior angles of a regular polygon

Activity 11.3

Get polygon cards.

Trace the edges of a regular pentagon and octagon on sheets of paper.

Extend the edges with a ruler and a pencil to form a straight line.

Using a protractor, measure the interior angles, then add to get their sum.

Then measure the exterior angles, add them to find their sum

What did you find out?

What is the objective of the activity?

Summary

The sum of the interior angles of a regular polygon is found by adding all the interior angles.

All the interior angles of a regular polygon are equal.

The sum of the exterior angles of a regular polygon is found by adding all the exterior angles.

All the exterior angles of a regular polygon are equal.

- For any regular polygon of **n sides**, the sum of all exterior angles of a regular polygon is 360°.
- The **sum of the interior angles** of a polygon with 'n' sides is calculated using the formula **(n-2) x 180 degrees**.
- The **sum of all exterior angles is 360 degrees**.

To find the value of **one interior angle = 180- (360/n) or** $\frac{(n-2) \times 180^0}{n}$

To find the value of **one exterior angle = 360/n.**

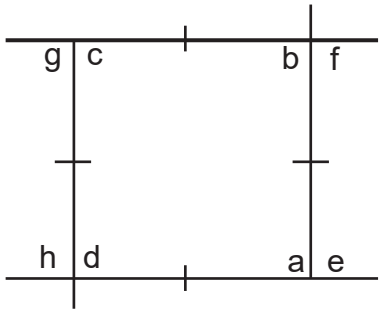
Example 1

Use a polygon card of a square and measure using a protractor to find its:

Sum of the interior angles.

Sum of the exterior angles.

Solution



The interior angles are a, b, c, d.

$\angle a = 90^o, \angle b = 90^o, \angle c = 90^o, \angle d = 90^o$

The sum of the interior angles is:

$= \angle a + \angle b + \angle c + \angle d$

$= 90^o + 90^o + 90^o + 90^o = 360^o$

Therefore, the sum of interior angles of a regular quadrilateral (square) is 360^o

The exterior angles are e, f, g, h.

$\angle e + \angle f + \angle g + \angle h$

$90^o + 90^o + 90^o + 90^o = 360^o$

Therefore, the sum of exterior angles of a regular quadrilateral (square) is 360^o.

Example 2

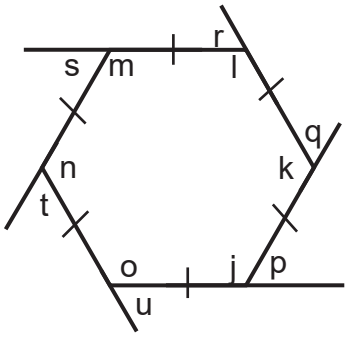
Trace from a regular polygon card.

Measure using a protractor to find its:

Sum of the interior angles.

Sum of its exterior angles.

Solution



The interior angles are j, k, l, m, n, o.

The sum of the interior angles is:

$= \angle j + \angle k + \angle l + \angle m + \angle n + \angle o$

$\angle j = 120^0, \angle k\ 120^0, \angle l = 120^0, \angle m = 120^0, \angle n = 120^o$ and $\angle o = 120^o$

$= 120^o + 120^o + 120^o + 120^o + 120^o + 120^o$

$= 720^o$

Therefore, the sum of interior angles of a regular hexagon is 720^0.

(b) **The exterior angles are p, q, r, s, t, u**

$\angle p + \angle q + \angle r + \angle s + \angle t + \angle u$

$= 60^o + 60^o + 60^o + 60^o + 60^o + 60^o = 360^o$

Therefore, the sum of exterior angles of a regular hexagon is 360^o.

Application activity 11.3

1. Using the polygon cards, draw and measure to find the sum of the interior angles and the sum of the exterior angles of the regular polygons below:

   a) Pentagon (b) heptagon (c) Nonagon (d) Hexagon

2. How do you find the value of the interior angle for each regular polygon given in (1)?

11.4 Finding the interior and exterior angles of a regular polygon

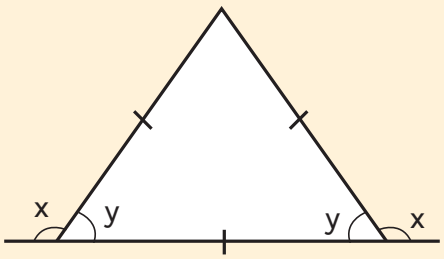
Activity 11.4

Study the sketch of a triangle.

Angle x is 120°, what is angle y?

Show your working out.

Present your findings to the class.



Summary

Interior and exterior angles of a regular polygon are supplementary.

That is, **exterior angle + interior angle = 180°**

Example 1

The exterior angle of a regular polygon is 72°. Find its interior angle.

Solution

Exterior angle + interior angle = 180°

72° + interior angle = 180°

72° – 72° + interior angle = 180° – 72

=Interior angle108°

Example 2

The interior angle of a regular polygon is 60degrees. Find its exterior angle.

Solution

Exterior angle + interior angle = 180.

Exterior angle + 60° = 180°

Exterior angle + 60° – 60° = 180° – 60°

= Exterior angle 120°

Application activity 11.4

1. Find the interior angle given the following exterior angles:
   (a) 76 degrees (b) 125 degrees (c) 105 degrees (d) 90 degrees
2. Find the exterior angle given the following interior angles:
   (a) 120 degrees (b) 45 degrees (c) 58 degrees (d) 135 degrees

11.5 Finding the sum of interior angles of a regular polygon

Activity 11.5

Draw a regular polygon of four sides (the square).

Measure its interior angles using a protractor.

Record your findings.

Try to measure its exterior angles.

Suggest how you can find the exterior angles while you have interior angles.

Now draw a regular pentagon.

Mark one vertex.

Draw straight lines connecting to other vertices.

What do you observe?

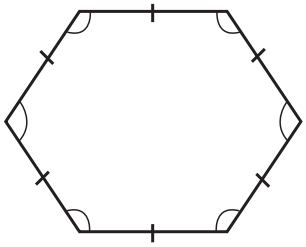
Summary

- Sum of interior angle is equal to each interior angle multiplied by the number of interior angles.
- The sum of interior angle of each triangle is 180°.
- Interior angle sum of a polygon = Number of triangles in a polygon x 180°.
- Sum of interior angle = $(n-2)\times 180^{\circ}$ where (n) is the number of sides.
- The number 2 is subtracted from the number of sides because 2 sides do not form a triangle.

See examples below:

Example 1:

Method 1



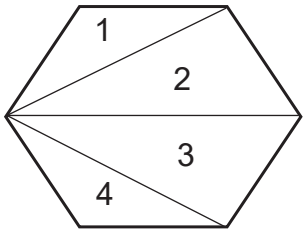
Find the sum of interior angles of a regular hexagon.

Solution

A hexagon has 6 sides. So, n = 6

Sum of interior angles $=(n-2)\times 180^{o}=(6-2)\times 180^{o}=4\times 180^{o}=720^{0}$

Method 2



1 triangle = 180°

4 triangles= 4 x 180° = 720°

The sum of interior angles of a regular hexagon is 720^{0}

Method 3

Each exterior angle= $\frac{360^{0}}{6}=60^{0}$

Exterior angle + Interior Angle = 180^{0}

Each interior angle = 180^{0} – Exterior angle = 180^{0}- 60 = 120^{0}. Therefore, the sum of interior angles = 120^{0} x 6 = 720 degrees.

Example 2

Find the number of sides of a polygon whose interior angle sum is 360^o

Solution

Sum of interior angles = $(n-2) \times 180^o$

$360^o = (n-2) \times 180^o$

$360^o = 180^o n - 360^o$

$360^o + 360^o = 180^o n$

$720^o = 180^o n$

$n = 4$

Therefore, the polygon has 4 sides. It is square.

Application activity 11.5

1. Find the sum of interior angles of the following regular polygons:
   (a) Octagon (c) Nonagon (d) Decagon
2. Find the sum of interior angles of regular polygons whose number of sides are:
   (a) 17 sides (b) 8 sides (c) 10 sides
3. Find the number of sides of polygons regular whose interior angles sum is:
   (a) 360^0 (b) 540^0 (c) 720^0

11.6 Exterior angles of regular polygons and their sum

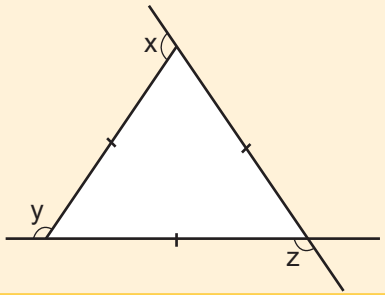
Activity 11.6

Use a protractor to measure angles x, y, and z.

Find the sum of the exterior angles.

What do you get?

Present your findings to the class.



Summary

The sum of exterior angles of a regular polygon is 360^o.

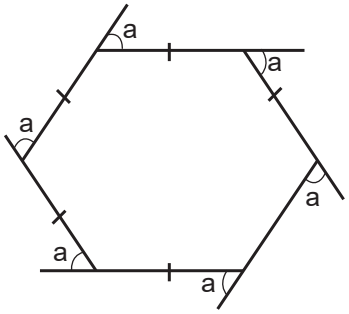
Number of sides = 360^o divided by one exterior angle.

$$\text{Exterior angle} = \frac{360^\circ}{n}$$

Where n= number of sides.

Example 1

Work out the value of a.



Solution

Sum of exterior angles is 360^o

$a + a + a + a + a + a = 360^o$

$6a = 360^o$

$\frac{6a}{6} = \frac{360}{6}$

$a = 60^0$

Example 2

Find the number of sides of a regular polygon with an exterior angle of 45^o. Name the polygon.

Solution

Each exterior angle = 45^o.

Exterior angle sum

$$\text{No. of sides} = \frac{\text{Exterior angle sum}}{\text{Each exterior angle}}$$

$$\text{No. of sides} = \frac{360^0}{45^0}$$

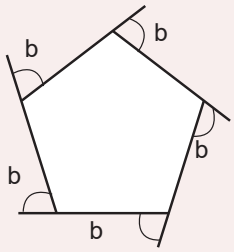
No. of sides = 8 sides

The polygon is an octagon.

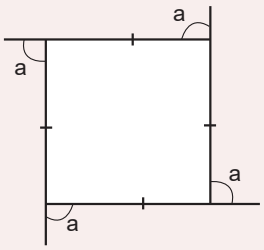
Application activity 11.6

1. Find the size of the exterior angles of the following regular polygons:

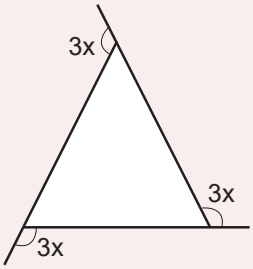
(a)



(b)



(c)



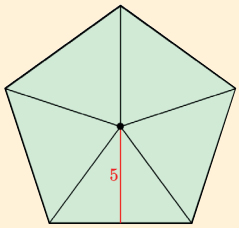
2. Find the number of sides of a regular polygon whose exterior angle is 60^o.
3. Find the exterior angle of each of the following regular polygons:
   a) Octagon b) Decagon c) Pentagon
4. The size of each interior angle of a regular polygon is twice its exterior angle.
   a) Find the exterior angle and interior angle
   b) Name the polygon
5. Find the exterior angle of a regular polygon whose interior angles is:
   (a) 120^o (b) 135^o

11.7 Finding perimeter of regular polygons

Activity 11.7

The perimeter of a shape is defined as the total length of its boundary.

Draw a regular pentagon and mark its centre.



Use a ruler to measure the length of one side of the pentagon.

Use a ruler to measure the perimeter of the pentagon and record it.

Compare the product **5 x side** and the perimeter you measured. Are they equal?

State the formula for finding perimeter of any regular polygon.

Present your findings to the class.

Summary

To find the perimeter of a regular polygon, add the lengths of its sides.

Perimeter is the distance round a shape or polygon.

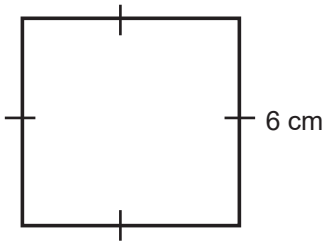
If the regular polygon has **n sides**, the perimeter P is $P = n \times side$.

Example 1

Find the perimeter of a square whose side is 6cm.

Solution

To find the perimeter (p) add all sides of the given polygon.



P = sum of all sides

P = 6cm + 6cm + 6cm + 6 cm = 4 x 6 cm

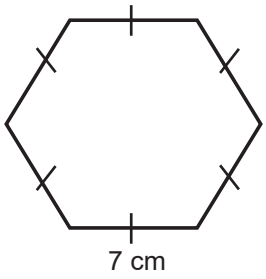
= 24 cm

Example 2

Find the perimeter of a hexagon whose side is 7 cm.

Solution

To find the perimeter (p) add all of the given polygon.



P= sum of all sides

P= 7 + 7 + 7 + 7 + 7 + 7 cm = 6 x 7cm

P= 42 cm

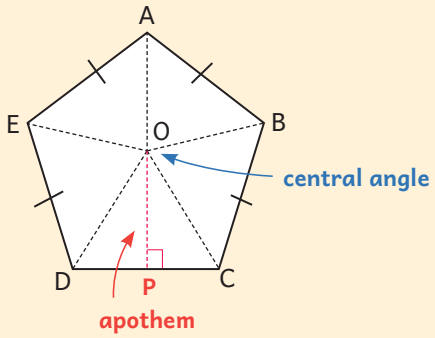
Application activity 11.7

1. Find the perimeter of an equilateral triangle with sides of 7 cm.
2. Find the perimeter of a regular hexagon of sides 5 cm.
3. Find the perimeter of a regular pentagon of sides 8 cm.
4. Find the perimeter of a regular heptagon with side 4 cm.

11.8 Finding area of regular polygons

Activity 11.8

Draw a regular pentagon of vertices AEDCB of side 16cm and center O.



Locate the middle P of the side DC of the polygon.

Draw a straight line from the centre O to the point P you located on the side DC.

Take a ruler and measure the length of OP. Is it perpendicular to the side DC?

The line OP drawn is called apothem. Can you use the apothem and the length of the side DC to find the area of the triangle ODC?

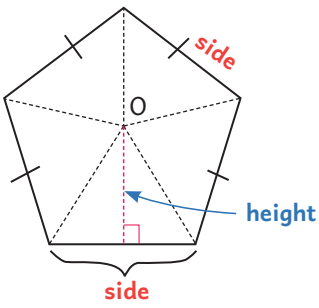
How many equal triangles are in the pentagon AEDCB?

Use the area of one triagle to find the total area of the pentagon AEDCB.

Discuss how you can get the area of any regular polygon.

Summary

The **apothem of a regular polygon** is the line segment from the center of the plygon and perpendicular to one of the sides. In a regular polygon, the apothem joins the centre of the polygon and the middle of the side.



When the side of the regular polygon is given, the apothem is considered as the height of one triangle of the regular polygon.

The regular polygon with **n sides** has **n equal triangles**.

The area of one triangle = $A_1 = \frac{1}{2} b \times h = \frac{1}{2} \times (side \times a) = \left(\frac{Side}{2} \times a\right)$

The total area of the polygon= $n \times \left(\frac{Side}{2} \times a\right) = \frac{1}{2} \times n \times side \times a$

Area of a regular polygon with n sides= $\frac{1}{2} \times n \times side \times a$ where a is the apothem.

Given that the product **n x side** is the **perimeter** of the regular polygon, p = n x side,

Area of a regular polygon with n sides = $\frac{1}{2} \times p \times a$ where a is the apothem and P is the perimeter.

Example 1

Find the area of a regular pentagon whose side is 6 cm and apothem is 4 cm.

Solution

$$\text{Area} = \frac{\text{Apothem} \times \text{Perimeter}}{2}$$

Given that Apothem = 4 cm and Side = 6 cm,

Perimeter = 6 cm+ 6 cm+ 6 cm+ 6cm + 6cm = 30 cm. Therefore,

$$\text{Area} = \frac{4cm \times 30cm}{2} = 60cm^2$$

Example 2

The side of a regular nonagon is 10 cm. Its apothem is 7 cm. Find its area.

Solution

A nonagon has 9 sides

Perimeter = 9 × sides

= 9 × 10 cm

= 90 cm

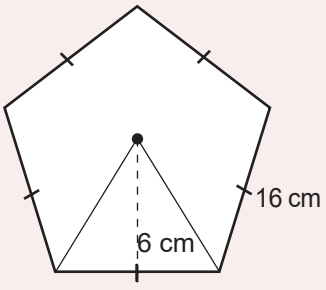
$$Area = \frac{1}{2} \times apothem \times perimeter$$

$$= \frac{7cm \times 90cm}{2} = 315cm^2$$

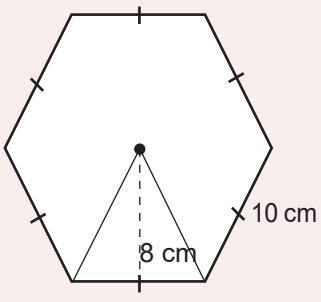
Application activity 11.8

1. Find area of an equilateral triangle whose side is 10 cm and apothem is 4 cm.
2. What is the area of a square garden with a perimeter of 24 m and apothem is 3 cm?
3. ABC is an equilateral triangle which has a side of 12 cm and apothem of 6 cm. Find its area.
4. Find the area of the following regular polygons:

(a)



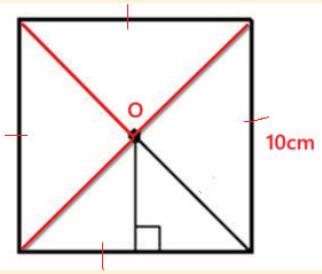
(b)



11.9 Finding sides and apothem of a regular polygon

Activity 11.9

Draw a square of side 10 cm.



What is its perimeter? What is its area?

Locate its center O and then, tell your colleague how you can get the length of its apothem.

Summary

Apothem is the line segment from the centre of a polygon perpendicular to the side of a polygon.

$apothem = \frac{2 \times area}{perimeter}$ where area is the area of the polygon, and perimeter of the polygon.

Side = Perimeter ÷ Number of sides

Example 1

Find the side of regular hexagon whose area is $108\,cm^2$ and 6 cm of apothem.

Solution

Area=$108\,cm^2$

Apothem = 6cm

Side=?

$$perimeter = \frac{2 \times area}{apothem} = \frac{2 \times 108cm^2}{6cm} = 36cm$$

$$side = \frac{perimeter}{number\ of\ side} = \frac{36cm}{6} = 6cm$$

Example 2

Find the apothem of a regular pentagon whose perimeter is 10 cm and the area is 60 cm^2.

Solution

perimeter = 10 cm. Area = $60\,cm^2$

Apothem = ? cm

$$apothem = \frac{2 \times area}{perimeter} = \frac{2 \times 60cm^2}{10cm} = 12cm$$

Application activity 11.9

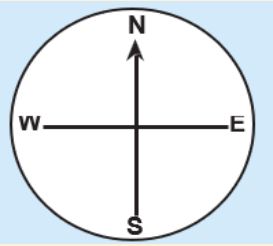
1. Differentiate between apothem and side of a regular polygon.
2. Draw a regular hexagon and show the apothem.
3. Find the apothem of a regular pentagon whose side is 8 cm and area is 120 cm^2.

4. Find the apothem of a regular hexagon whose side is 12 cm and area is 144 cm^2.
5. Find the apothem of a regular octagon whose side is 10 cm and area is 160 cm^2.
6. What is the apothem of a regular decagon with a side of 8 cm and area of 240 cm^2.

11.10 Finding direction and compass points

Activity 11.10

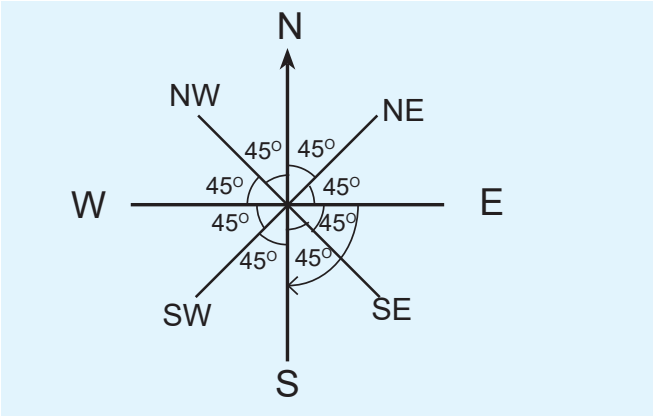
- Get a pair of compasses, a ruler, pencil and a real compass.
- Observe the compass and identify directions (N,S,W,E)
- Draw a circle.
- Draw a compass direction inside the circle.
- Name the 4 cardinal points as observed on the compass.
- Measure the angles between the lines of compass direction.



Summary

A compass shows direction.

The cardinal points of a compass have 90 degrees between them.
A compass has angle sum of 360°. It has 8 angles.



Each angle of a compass = $\frac{360^0}{8} = 45^0$

Clockwise turn is a right turn and **anti-clockwise** turn is the left turn.

To find direction of one point from another, indicate North on starting point, finally show the point which is wanted.

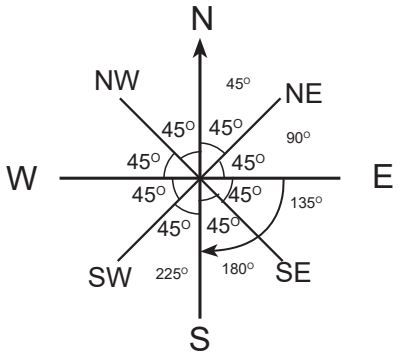
- When the turn (clockwise or anti-clockwise) is not given, turn clockwise.
- When the point is in North, start from North. That is, **N angle E** or **N angle W**.

While if the point is in South, start from South. That is **S angle E** or **S angle W**.

Example 1

Face East and turn clockwise through 90^0. What is the new direction?

Solution

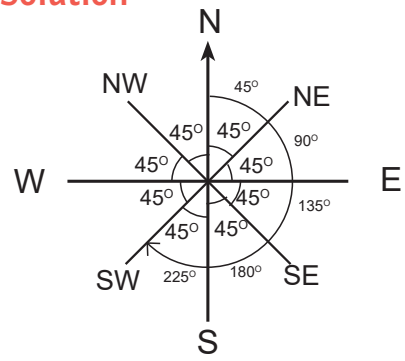


The new direction is **South**.

Example 2

Face North and turn clockwise direction through 225^0. What is the new direction.

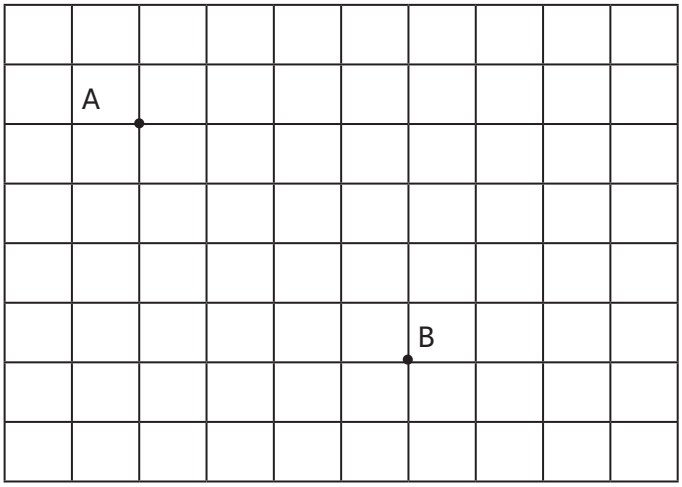
Solution



The new direction is **South-west**.

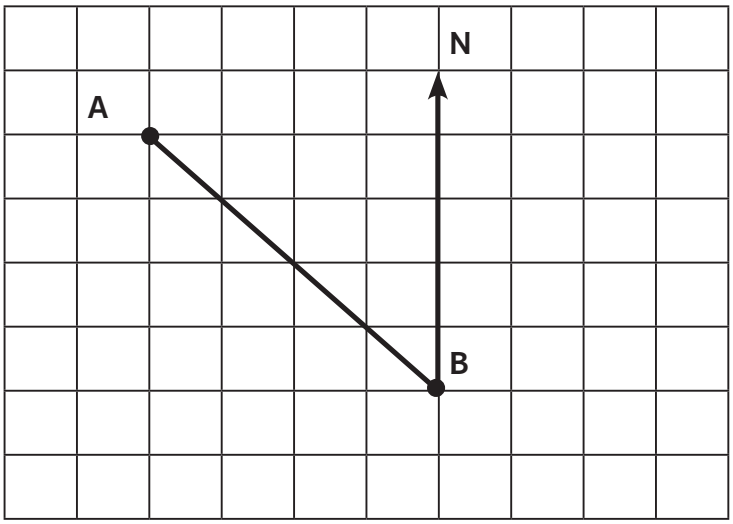
Example 3

What is the direction of A from B?

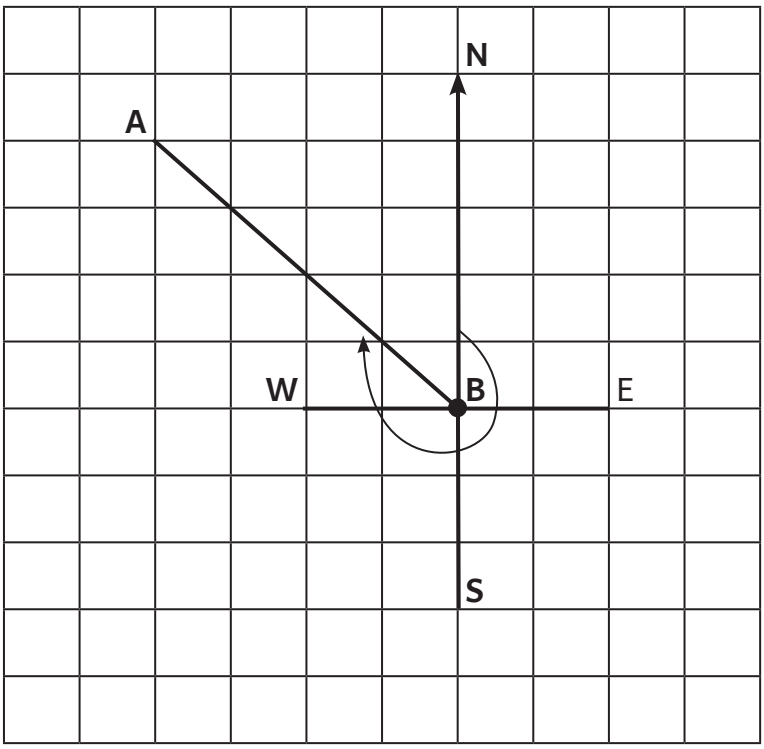


Solution

Step 1: join the points and indicate north on starting point (B) to find out A



Step 2: allocate other directions regarding on north of B then find the direction of A



Therefore, the direction of A from B is **North-West**

Application activity 11.10

Find the new direction.

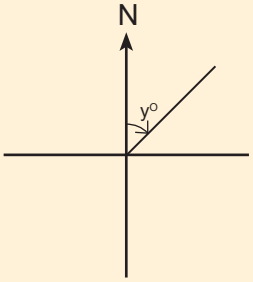
a) Clockwise 135° from West.
b) Clockwise 180° from North-East.
c) Anti-clockwise 90° from East.
d) Anti-clockwise45° from North.

11.11 Finding the bearing

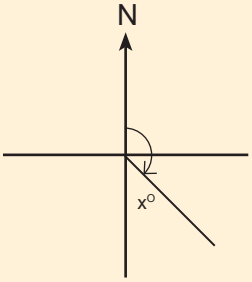
Activity 11.11

Measure the following angles using a protractor

(a)



(b)



Record and discuss your findings to the class.

Summary

To get the bearing, face North and only turn clockwise. Find the total angle of turn.

The cardinal points of a compass have 90 degrees between them.

The second cardinal points of a compass have 45 degrees between them.

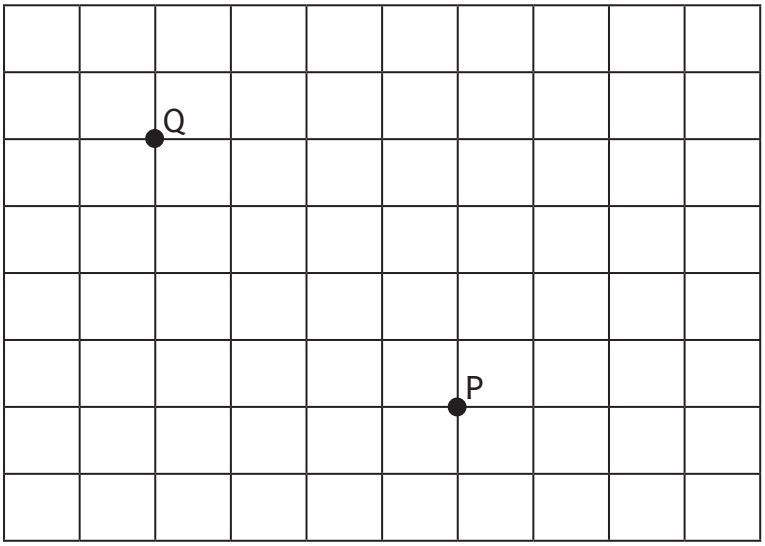
To find bearing of one point from another, indicate **north on starting point**, next show the point which is wanted.

The angle is written with 3 digits. For example, $10^{o} = 010^{o}$.

Degrees of bearing do not involve cardinal points, e.g :060^{o} not N60^{o}E

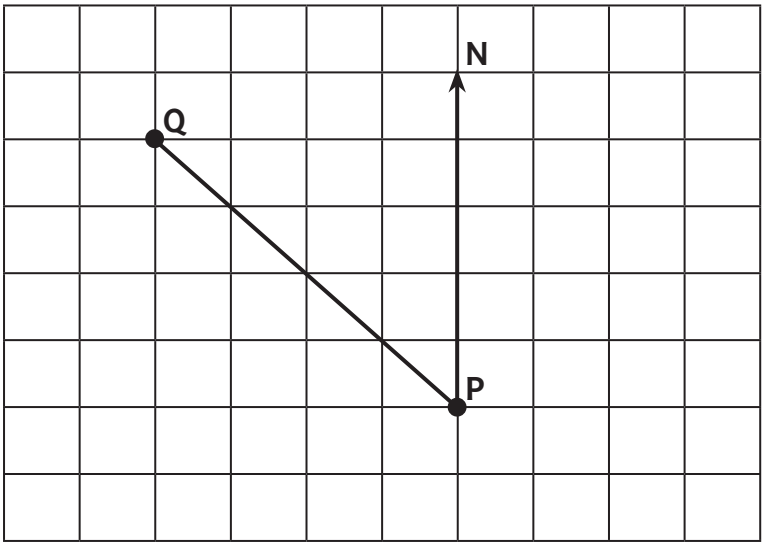
Example

What is the bearing of Q from P?

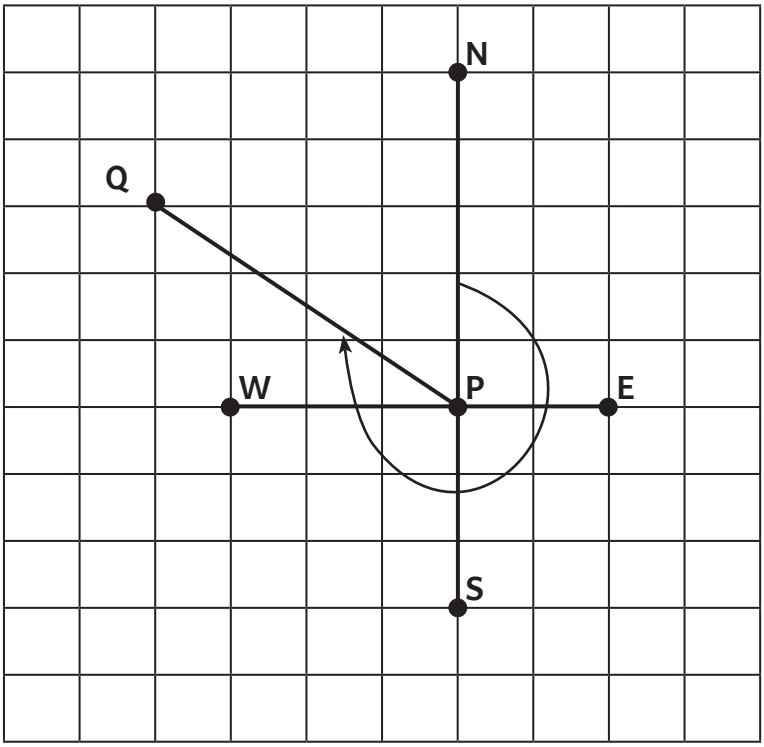


Solution

Step 1: Join the points and indicate north on starting point (P) to find out Q



Step 2: Allocate other directions regarding on north of P then measure the total angle of Q from of N allocated on P. The cardinal points of a compass have 90 degrees between them.



From N to E=90^0 From E to S =90^0

From S to W=90^0 ; From W to Q =40^0

Total angle =310^0

Therefore, the bearing of Q from P is **310^0**

Note: The bearing of P from Q is the reverse bearing = given bearing ± 180 degrees

= $310^0 + 180^0 = 490^0$

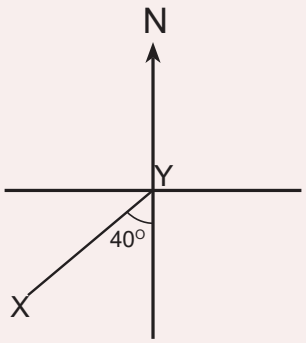
= $490^0 - 360^0 = 130^0$

The bearing of P from Q is 130^0

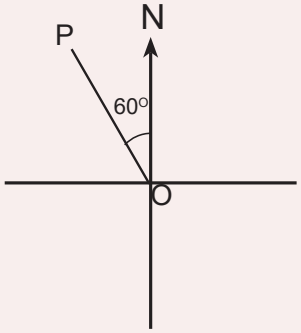
Application activity 11.11

Find the bearing of:

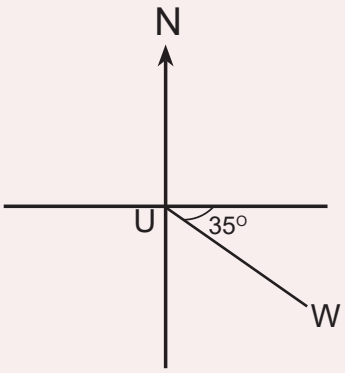
(a) X from Y



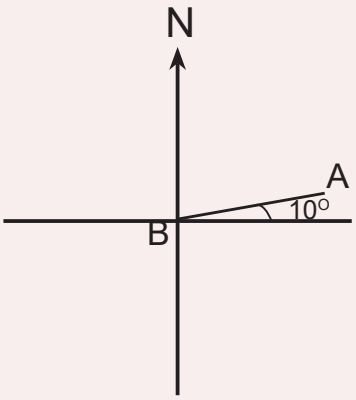
(b) P from O



(c) W from U



(d) A from B



11.12 Tiling in construction

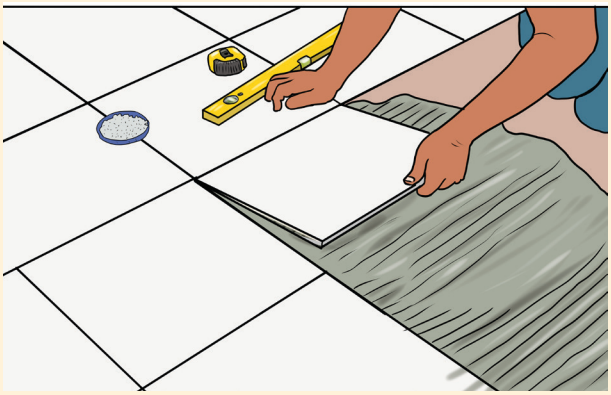
Activity 11.12

Get sheets of paper.

Use different polygon cards prepared for the lesson.

Using glue, fix suitable polygon cards in a pattern such that no gaps are left in between.

a) Name the activity you are carrying out.



b) Which polygon cards can be used to tile the plane? (Rectangular sheet)

c) Visit workers on tiling activity and ask them how they do things.

d) How is tiling important in daily life?

e) Present your working out to class.

Summary

Tiling is done to cover plane surfaces like floors, walls, compounds with tiles to form beautiful designs. Square, equilateral triangular, regular pentagonal, hexagonal and other shapes of tiles are used for tiling.

Tiling a plane requires the dimensions of the tiles to fit perfectly the area of the plane.

Designers measure the sides of a plane to calculate the total area to cover and they

compare it with the size of one tile to determine the number of tiles to be used. Use the following steps to determine the number of tiles needed to cover a room:

Step 1: Measure the Surface Area to be Covered

- For rectangular surfaces (e.g., floors, walls): Surface Area=Length×Width
- For irregular shapes: Divide the area into smaller rectangles, calculate each, and sum them up.

Step 2: Determine the Area of One Tile. If the tile is square or rectangular:

- Tile Area = Tile Length x Tile Width
- If the tile is triangular, hexagonal, or another shape, use the appropriate area formula.

Step 3: Calculate the Number of Tiles Needed

- Number of Tiles =Total Surface Area/Area of One Tile
- Round up to the nearest whole number since you can't use a fraction of a tile.

Step 4: Account for Wastage. As some tiles can be broken, you can add 10–15% extra tiles for cuts, breakages, and future repairs.

- Total Tiles = Number of Tiles×1.10.

Note: the tiles for wastage are counted only when it is recommended. Otherwise, do not count them.

Example 1

A square floor measures 9 m by 9 m. A builder tiles the floor with square tiles of sides 30 cm each.

a) Explain how he would carry out tiling.

b) How many tiles does he use?

Solution

a) He should lay the tiles in straight lines across the floor. He should fit the edge of one tile exactly to fit the edge of the other tile, along all edges. The floor should have a squared pattern/design.

b) Along one side of the room he uses:

= Length of room / Length of tile = $\frac{9\times100\text{cm}}{30\text{cm}}$ = 30 tiles

Along the other side of the room, he lays the same number of tiles, i.e, 30 tiles.

Altogether the tiles used are: side x side = (30 x 30) tiles = 900 tiles.

The interior angles help the builder to fix the joint perfectly.

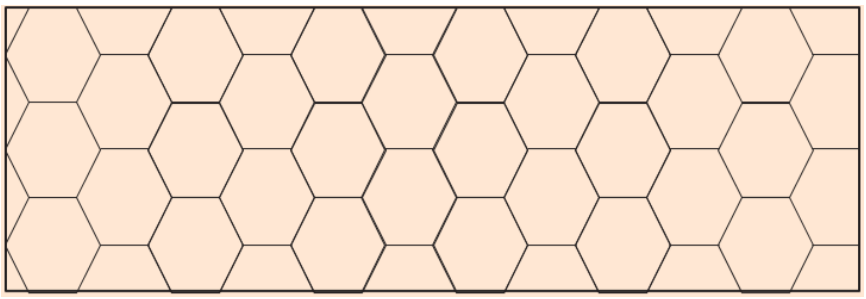
Example 2

Draw a tiling pattern for regular hexagonal pavers each of sides 1 cm and interior angle 120°.

Solution

Use a ruler to measure 1 cm sides and protractor to measure interior angle of 120°.

Fix the hexagon firmly along the 120^0 vertices



Application activity 11.12

1. Use regular pentagonal cards of sides 3 cm to tile a plane sheet of paper. How many cards do you use?
2. Use regular hexagonal cards of sides 2 cm. Tile your table top or desktop.
3. Visit a construction side or buildings. Study the tiling.
   a) What polygons were used?
   b) Copy the tiling design in your books.
4. A room had a floor measuring 9 m by 6 m. A builder laid tiles on the floor measuring 30 cm by 30 cm. How many tiles did the builder lay?
5. A mason laid hexagonal pavers on a compound. Each paver was 40 cm across. The compound was 40 m by 20 m. How many pavers were laid on the compound?

11.13. End of unit assessment

Assessing Knowledge and understanding

1. Define

a) What is a regular polygon?

b) What is the formula for the sum of interior angles of an 5-sided polygon?

2. State:

a) The bearing of point B from point A is 045°. What does this mean?

b) How many sides does a heptagon have?

3. What is the angle between North and North-East?
4. Find the sum of interior angles of the following regular polygons:

(a) Decagon (b) heptagon (c) Octagon

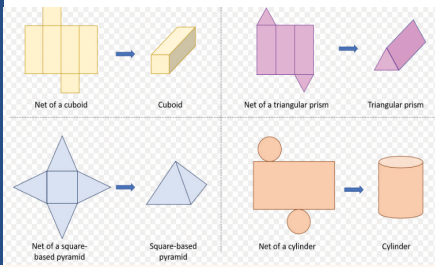
Assessing skills

5. Find the number of sides of regular polygons whose interior angle sums are:
   (a) 540° (c) 720°
6. The perimeter of a regular pentagon is 120 cm. How long is its side?
7. A regular decagon of a side 12 cm has apothem of 18 cm. Calculate its area.
8. The interior angle sum of regular polygon is 180°. Find its number of sides.
9. A square room has sides 4 cm. Square tiles of sides 40 cm were laid on its floor. How many tiles were laid?
10. Kibeho is on a bearing of 220° from Busasamana. Find the bearing of Busasamana from Kibeho?

Assessing attitudes and values

11. Why is understanding bearings important for pilots or ship captains? Give two reasons.
12. a) Surveyors use bearings to map land. How might errors in bearings affect property boundaries?
    b) Floor tiles are often regular polygons. Why might hexagons be better than squares for some designs?

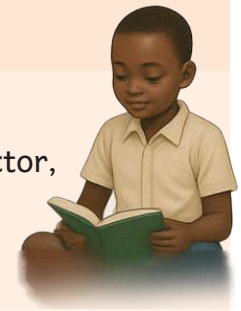
UNIT 12



CONSTRUCTION OF POLYGONS AND NETS FOR CUBOIDS AND PRISMS

Key unit competence: You will be able to construct polygons and design nets to make cuboids and prisms.

Learning objectives: By the end of this unit, you should be able to:



Knowledge and understanding:

- Show how to construct polygons with given properties using a protractor, a ruler and a pair of compasses.
- Demonstrate how a 2D shape can be folded to make a 3D solid and name the 2D shape used.

Skills:

- Construct polygons using a protractor, a ruler and a pair of compasses.
- Explore different strategies for constructing polygons with given properties.
- Design nets to make cuboids and prisms.

Attitudes and values:

Appreciate that there are different successful approaches to accurately construct a polygon or design a net.

12.0 Introduction

In this unit, you will learn how to **draw and construct different polygons**, including squares, rectangles, pentagons, and hexagons, using accurate measurements. You will also explore **nets** flat, 2D shapes that fold into 3D objects like cuboids and prisms. By working with rulers, protractors, and compasses, you will develop skills in geometric construction while understanding the properties of these shapes.

These concepts have many **real-life applications**: designing boxes for packaging objects. Architects use nets to create models of houses, and engineers rely on polygonal shapes to construct bridges and furniture.

Introductory Activity

Observe the following objects: stop signs for the road, tiles and boxes.

"What shapes do you see? How are they formed?"

Which ones have equal sides and equal angles?

Can you draw them? Which materials can you use to draw them?

12.1 Drawing triangles using a protractor and ruler

Activity 12.1

Use the following line AB

Use the ruler to draw a right angle at A using the line AC.

Join the point B and C.

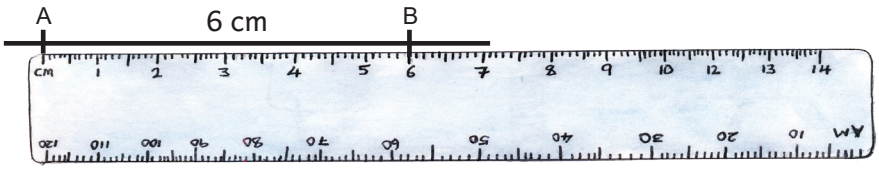
What shape do you get?

Summary

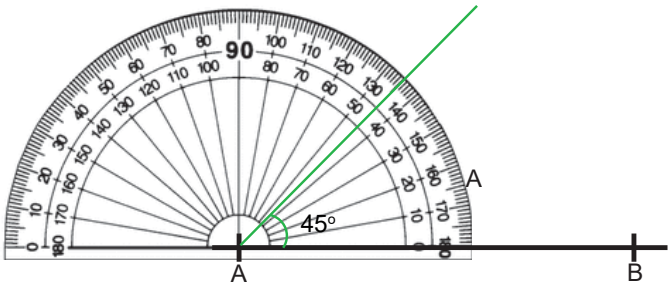
Drawing a triangle

Using a protractor and ruler, we are going to draw a triangle ABC with AB = 6 cm, AC = 4 cm and angle BAC = 45^0.

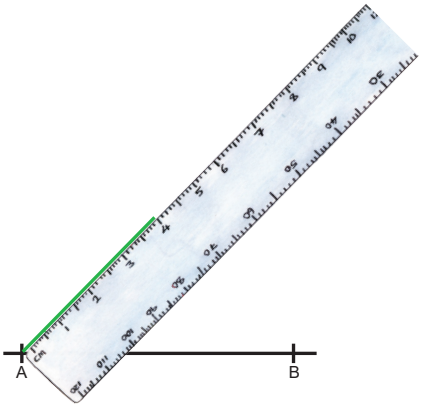
Step 1: Draw a baseline and mark on it line segment AB (6 cm) apart using a ruler.



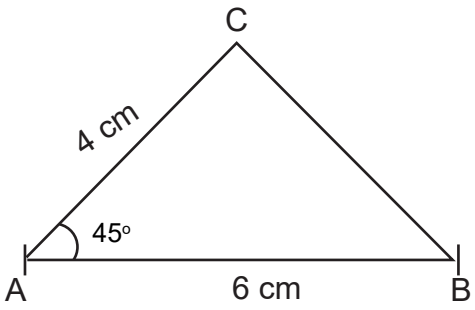
Step 2: Measure angle BAC = 45^0 using a protractor.



Step 3: Measure 4 cm of the 45^0 line from A. Use a ruler and label it C.



Step 4: Join B to C. Measure BC and name the triangle ABC.



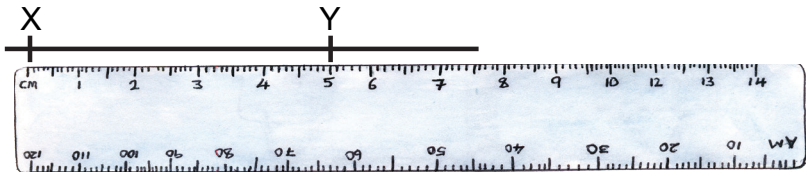
BC=5.91cm

Example

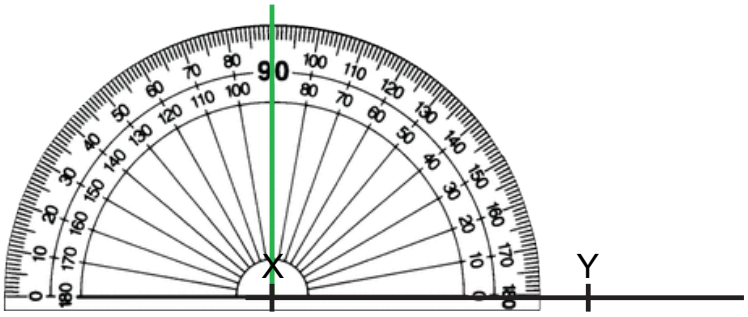
Draw triangle XYZ with XY = 5 cm, $\angle ZXY = 90^0$ and $\angle XYZ = 60^0$. Use a ruler and a protractor.

Solution

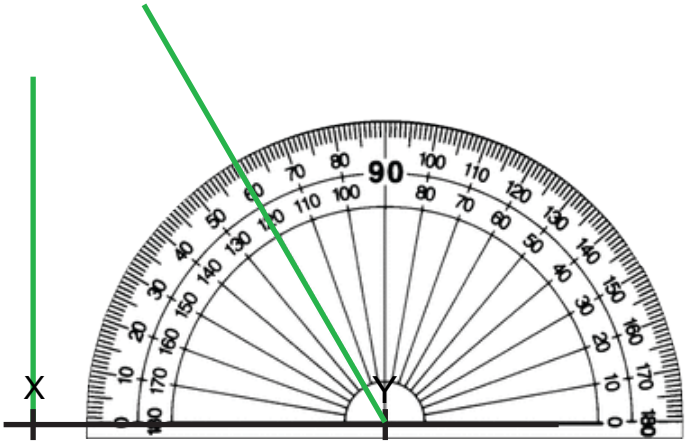
Step 1: Use ruler to draw a baseline and mark on it line segment XY = 5 cm apart.



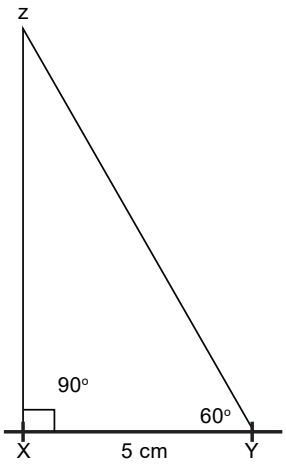
Step 2: Use a protractor to measure $\angle 90^0$ at X.



Step 3: Use a protractor to measure $\angle 60^0$ at Y.



Step 4: Join the 90^0 line to 60^0 line. Label the meeting point Z. Name the ΔXYZ .



Application activity 12.1

Using ruler, protractor and a pencil, construct the following triangles:

1. ABC with AB = 6 cm, $\angle ABC =$ 60^0 and BC = 4cm.
2. PQR with P = 50^0, Q = 30^0 and PQ = 7 cm.
3. MNO with $\angle$M = $\angle$N = 50^0 and MN = 5 cm.
4. ABC with AB = 7.2 cm, AC = 5.5 cm and $\angle CAB$ 75^0.

12.2 Drawing a square using a protractor and ruler

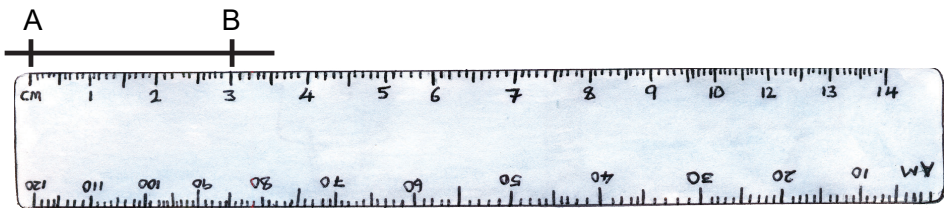
Activity 12.2

- What are the main characteristics of a square: how many sides? Do they have the same length? How many right angles?
- Try to use a protractor and a ruler to draw a square of side 4cm on the manila paper.
- Use a protractor to check that all angles are 90^0.

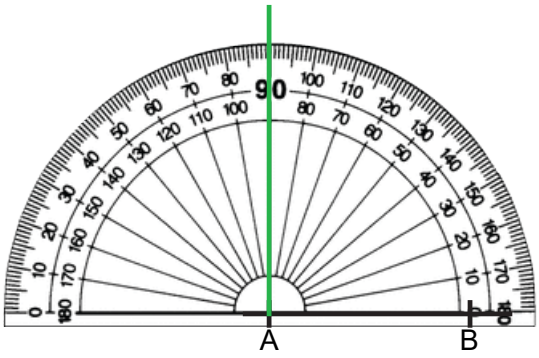
Summary

Let us use a ruler and a protractor to draw a square ABCD with sides 3 cm.

Step 1: Use a ruler to draw a baseline AB with 3 cm apart

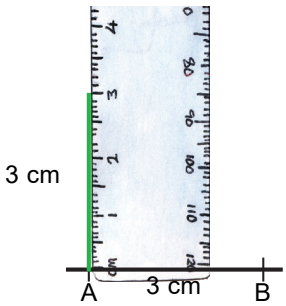


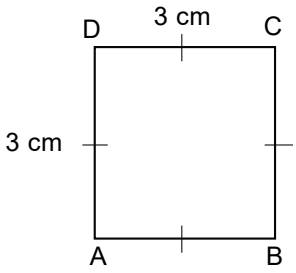
Step 2: Using protractor, measure $\angle 90^0$ at A and B



Step 3: From A, measure 3 cm along the 90^0 line. Label it D. Do the same at B and label it C.

Step 4: Join D to C and name the square ABCD.





- A square has 4 equal sides.
- Angles of a square measure 90^0.

Application activity 12.2

Use ruler and protractor to draw the following:

a) Square PQRS of side 5.5 cm.

b) Square ABCD of side 4 cm.

c) Square DEFG of side 3.8 cm.

d) Square ABCD of side 4.2.

12.3 Drawing a rectangle using a protractor and ruler

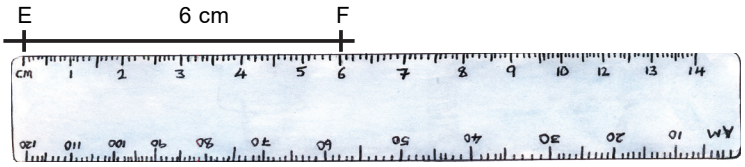
Activity 12.3

- What are the main characteristics of a rectangle? How many sides? Do they have the same length? How many right angles does a rectangle here?
- Try to use a protractor and a ruler to draw a rectangle of length 8cm and width 4cm on the manila paper.
- Use a protractor to check that all angles are 90^0.

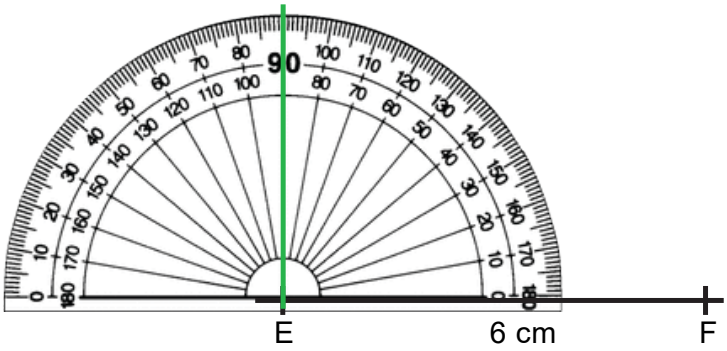
Summary

Let us use a protractor and a ruler to draw a rectangle EFGH with length 6 cm and width 4 cm.

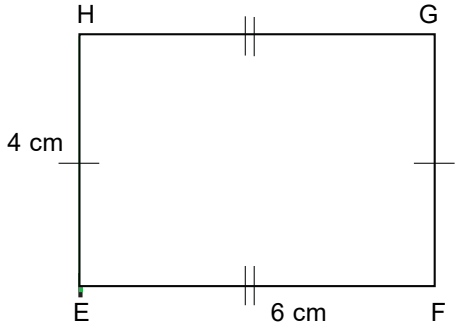
Step 1: Use ruler to draw a baseline EF 6 cm apart.



Step 2: Using protractor, measure $\angle$ 90^0 at E and F. Draw a straight line from E through the 90^0 mark.



Step 3: From E, measure 4 cm along the 90^0 line. Label it H. Do the same at F and label it G.
Step 4: Join H to G with a straight line.



Measure $\angle H$ *and* $\angle G$. Each must be equal to 90^o. Measure line HG. It must be equal to 6 cm.

Denote the angles with 90^0 by the symbol (⌐). Denote equal opposite sides with equal angles.

In a rectangle,

- Two opposite sides are equal: EH = FG and EF = HG.
- Each angle of a rectangle measures 90^0.

Application activity 12.3

Use ruler and protractor to draw the following:

a) Rectangle ABCD with sides AB = 5.7 cm and BC = 3 cm.

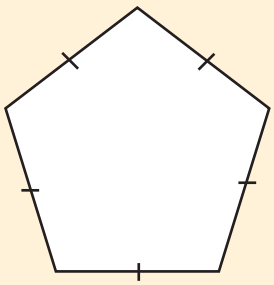
b) Rectangle HIJK with sides HI = 5.6 cm and IJ = 3 cm.

c) Rectangle LMNO with sides LM = 6 cm and MN = 4.2 cm.

12.4 Drawing a regular pentagon using a protractor and ruler

Activity 12.4

Study the figure shown and answer questions that follow.



- Name the figure if all sides have the same length.
- Measure its exterior angle.
- Measure its interior angle
- Can you draw it using a ruler and a protractor?
- Explain how you can draw it.

Example 1

Use a protractor and ruler to draw a regular pentagon PQRST with side 3 cm.

Solution

Step 1: Find the interior angle. To get the interior angle, first find the exterior angle.

$$\angle exterior = \frac{360^0}{number\ of\ sides}$$

$$= \frac{^{72}\cancel{360^0}}{^{1}\cancel{5}} = 72^0$$

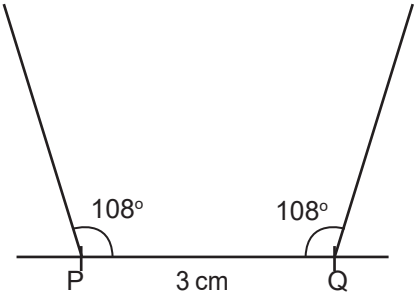
$$= 72^0$$

$$\angle interior = 180^0 - exterior\ angle$$

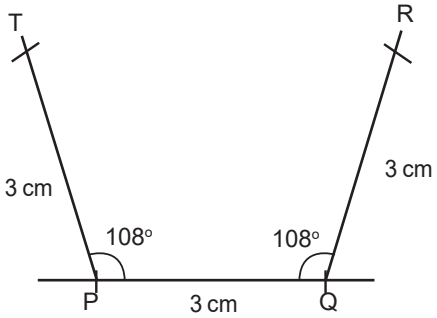
$$= 180^0 - 72^0$$

$$= 108^0$$

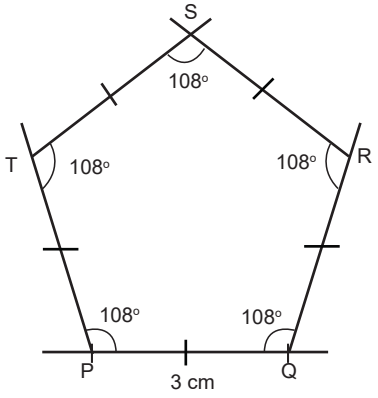
Step 2: Draw the baseline and mark off PQ (3 cm) apart. Use a protractor and draw 108° at P and Q



Step 3: From P, measure 3 cm along the 108^0 line. Label it T. Do the same at Q and label it R.



Step 4: Draw 108^0 at T and R. Label the point where the two-line meet S. Name the pentagon PQRST. Then indicate that all sides are equal.



To construct a regular polygon (pentagon, hexagon and other regular polygons),

- First find the exterior angle then the interior angle.
- Always start with the baseline.

- Use the interior angle to draw the second side;
- Continue to the next side keeping the equal sides and equal interior angles.

Application activity 12.4

Using a protractor and a ruler, draw the following regular pentagons:

(a) Pentagon ABCDE of side 5 cm.

(b) Pentagon BCDEF of side 5.5 cm.

(c) Pentagon PQRST of side 6.5 cm

12.5 Drawing a regular hexagon

Activity 12.5

- Pick a pencil, a protractor, ruler and a sheet of paper.
- Find the value of interior angle of a regular hexagon.
  - Draw the horizontal line segment AB of 6 cm.
  - From B, measure the angle ABC with the value equal to the interior angle of the regular hexagon such that BC = 6 cm.
  - Continue the process at C until you find a closed shape.

What shape do you find?

- What should you do to draw a regular six-sided figure?
- Discuss and share with other groups.

Summary

Let us use a protractor and a ruler to draw a regular hexagon STUVWX with side 3.5 cm.

Step 1: Find the interior angle. To get the interior angle, first find the exterior angle.

$$\angle exterior = \frac{360^0}{number\ of\ sides}$$

$$= \frac{^{60}\cancel{360^0}}{\cancel{6}_{1}} = 60^0$$

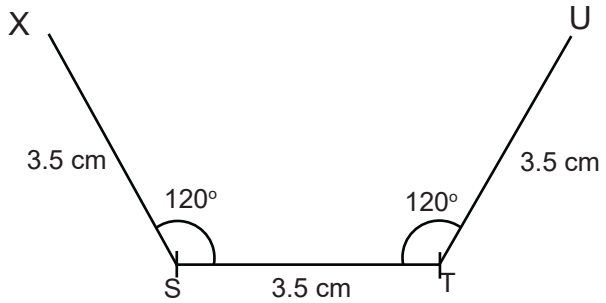
$$\angle interior = 180^0 - exterior\ angle$$

$$= 180^0 - 60^0$$

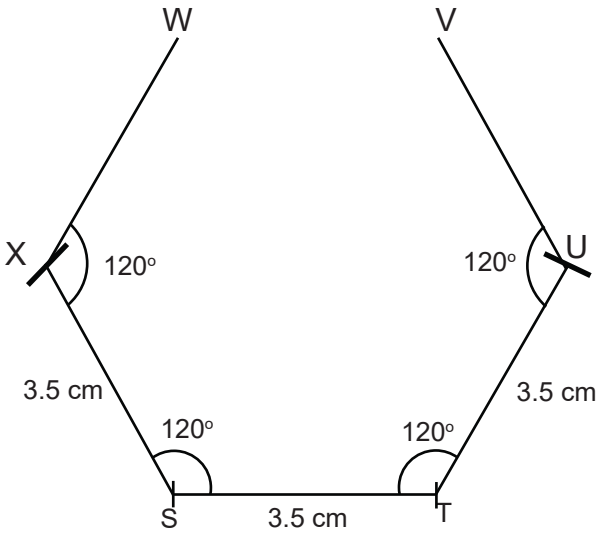
$$= 120^0$$

Step 2: Draw baseline ST 3.5 cm apart. Use a protractor to draw 120^0 at S and T.

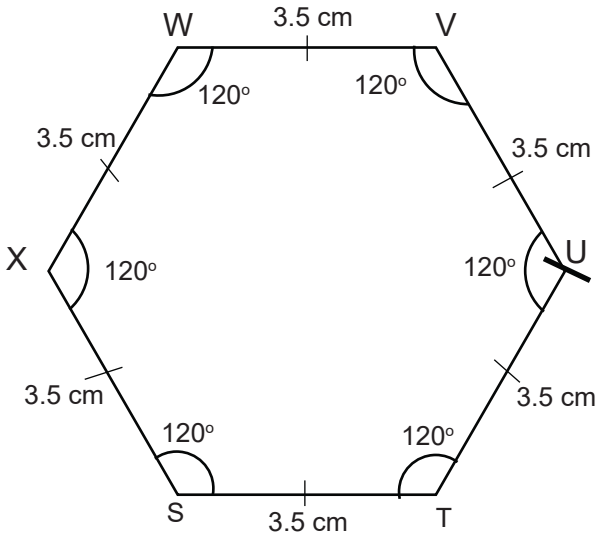
Measure 3.5 cm from S and T along the 120^0 lines. Label the point X and U.



Step 3: Draw 120^0 at X and U. Measure 3.5 cm from X and U along the 120^0 line. Label the points W and V.



Step 4: Draw a straight line joining W and V. Measure $\angle$W and $\angle$V to confirm they are 120^0 each. Indicate that all sides are equal. Name the polygon.



- A regular hexagon has 6 equal sides.
- A regular hexagon has 6 interiors equal angles. Each interior angle of a regular hexagon is 120^0.
- It has 6 exterior angles, each exterior angle of a regular hexagon is 60^0.

Application activity 12.5

Using a protractor and ruler, draw the following regular hexagons:

a) Hexagon ABCDEF of side 4 cm.

b) Hexagon GHIJKL of side 4.2 cm.

c) Hexagon KLMNOP of side 5 cm.

12.6 Constructing triangles using a pair of compasses and a ruler

Activity 12.6

- What's is an equilateral triangle?
- What is the interior angle of the equilateral triangle?
- Using ruler and compasses only, construct an equilateral triangle with side 3 cm.
- Present your working out to the class.

Summary

To construct a triangle, proceed as follow:

- To construct an equilateral triangle, measure and draw the baseline, then mark the same distances from both ends of the baseline to the top common vertex.
- To construct a triangle with all the sides given, measure and draw the baseline accurately. Construct the other two sides using a pair of compasses.
- To construct a triangle with 2 sides and 1 angle, construct in the order side, angle, side (SAS).
- To construct a triangle with 1 side and 2 angles, construct in the order side, angle, angle (SAA).

Let us use a ruler and compasses only to draw a triangle.

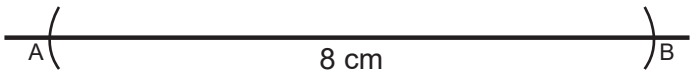
Example 1

Construct triangle ABC with $\overline{AB}$ = 8 cm, $\overline{BC}$ = 7 cm and AC = 5 cm. Use ruler and compasses only.

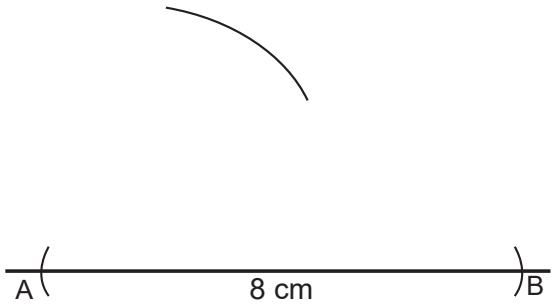
Solution

Step 1: Draw a baseline. Using a compass, measure 8 cm from ruler.

- Place a compass point at one side of the baseline.
- Make an arc along the baseline.
- Place the compass point at the drawn arc.
- Without adjusting the radius, draw an arc along the base line to the other side. Label the arcs A and B.



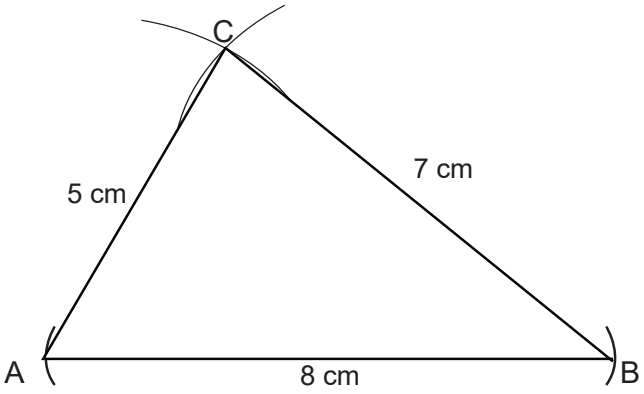
Step 2: Measure 5 cm from ruler. Place the compass point at A and make an arc above line AB.



Step 3: Measure 7 cm from ruler. Place the compass point at B and make an arc above AB to cross the arc down first.

Step 4: Label the point where the arcs cross each other C. Join A to C and B to C.

Name the triangle ABC.

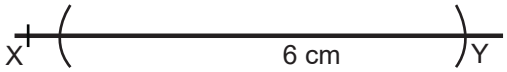


Example 2

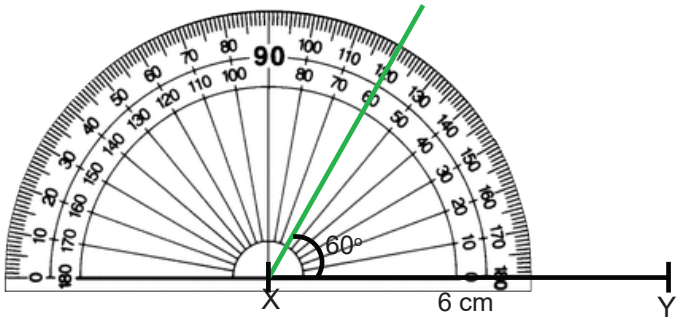
Construct triangle XYZ with $\overline{XY}$ = 6 cm, $\angle$ X 60^0 and $\overline{XZ}$ = 3 cm.

Solution

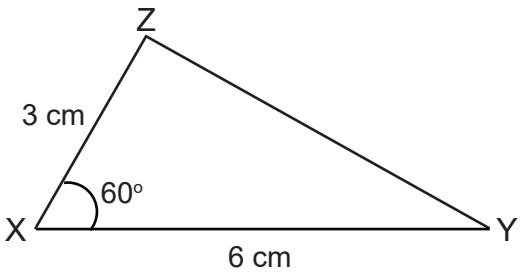
Step 1: Draw a baseline. Using a compass, measure 6 cm from the ruler. Place a compass point at one side of the baseline. Make an arc at the other side. Place the compass point at the drawn arc. Without adjusting the radius, draw an arc at the other side. Label the arcs X and Y.



Step 2: Construct $\angle 60^0$ at X. Measure 3 cm from the ruler. Place the compass point at X and make an arc on the 60^0 line.



Step 3: Join Y to the marked arc on the 60^0 line. Label the point Z. Name the triangle XYZ. Measure YZ.



Application activity 12.6

Using compasses and ruler only, construct the following triangles:

1. Equilateral triangle ABC of sides 4.5 cm.
2. Triangle PQR with PQ = 6 cm, QR = 4 cm and PR = 3 cm.
3. Triangle XYZ with XY = 5.5 cm, $\angle$ Y = 80^0 and YZ = 3.5 cm.
4. Triangle CDE with CD = 7.5 cm, angle C = 90^0 and angle CDE = 45^0.
5. Triangle JKL with JK = 7 cm, KL = 6 cm and JL = 5 cm.

12.7 Constructing a square using a pair of compasses and a ruler

Activity 12.7

- What is a square? How are sides and angles of a square?
- Draw line segment AB of 5 cm.
- Construct 90^0at A and another at B.
- Measure 5 cm from A along the 90^0 line. Label it as point D.

- Measure 5 cm from B along the 90^0 line. Label it C.
- Draw the top side of 5 cm connecting D to C. Name the figure formed.

Summary

- A square has 4 equal sides.
- A square has 4 right angles.

We can use a well sharpened pencil and a properly fixed pair of compasses to construct a square.

Example

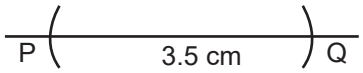
Construct a square of side 3.5 cm.

Solution

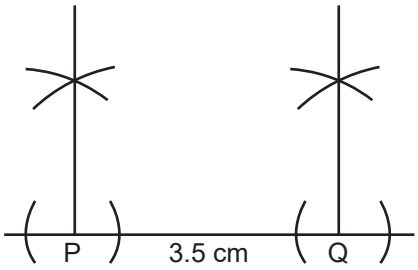
Step 1: Draw a line.

Measure 3.5 cm from the ruler.

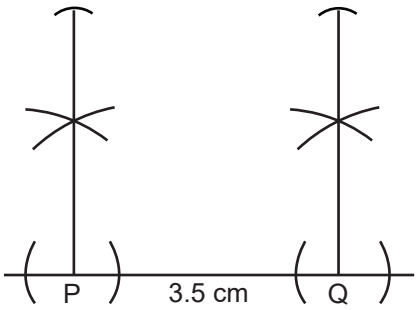
Mark off two points 3.5 cm apart. Name the points P and Q.



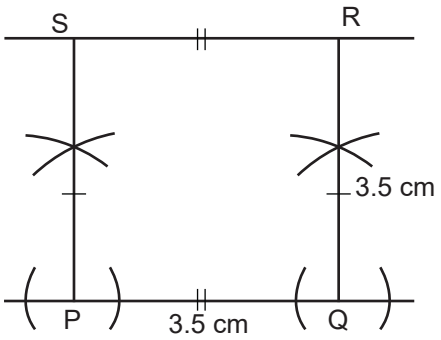
Step 2: Construct perpendicular lines at P, then at Q.



Step 3: Measure 3.5 cm from the ruler using compasses and a pencil. Mark PS 3.5 cm and QR 3.5 cm.



Step 4: Join R to S. Name points R and S.



Application activity 12.7

1. Construct squares with side given below:
   (a) 3 cm (b) 4 cm (c) 5.5 cm (d) 7.2 cm
2. Construct a square with a side of 5.6 cm.

12.8 Constructing a rectangle using a pair of compasses and a ruler

Activity 12.8

What is the rectangle? How are angles of a rectangle?

- Draw line segment XY of 5 cm.
- Construct 90^0 at X and another at Y.
- Measure 2 cm from X along the 90^0 line. Label it point D.
- Measure 2 cm from Y along the 90^0 line. Label it C.
- Draw the top side of 5 cm connecting D to C. Name the figure formed.

Summary

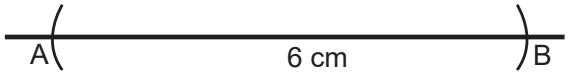
- A rectangle is an irregular polygon. Two parallel sides are equal but all angles are 90 degrees.
- To construct a rectangle, draw the baseline, construct the perpendicular widths and finally draw the opposite length.
- Use a sharp pencil and a properly fixed compass.

Example

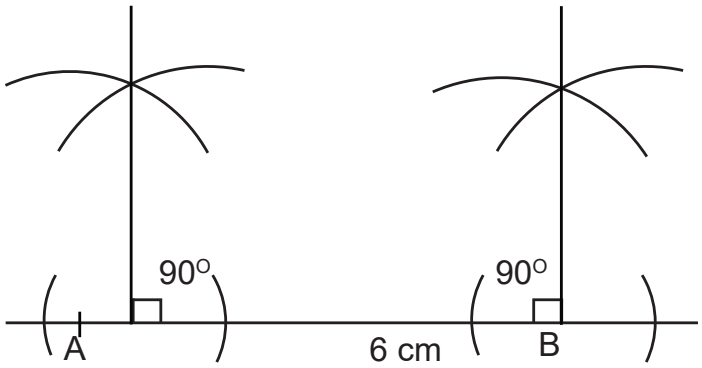
Use ruler and compasses to construct a rectangle ABCD of AB = 6 cm and BC = 4 cm..

Solution

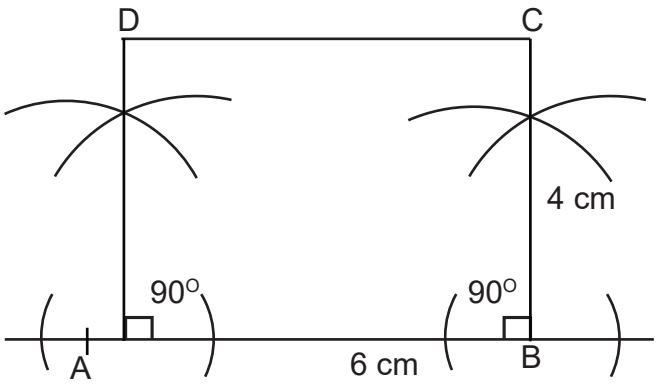
Step 1: Draw a baseline. Mark AB, 6 cm using compasses.



Step 2: Construct 90^0 at A and B. Now use a ruler to draw lines starting from A passing through the arcs. Draw another line from B through the arcs from B.



Step 3: Use a compass to measure and label points D and C from A and B of 4 cm respectively. Join D to C and name the rectangle ABCD.



Application activity 12.8

Using compasses and ruler only, construct the following rectangles:

a) Rectangle ABCD with length 5 cm and width 4 cm.

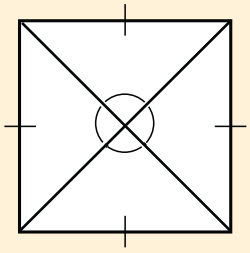
b) Rectangle WXYZ with length 8 cm and width 5 cm.

c) Rectangle ABCD with sides 5 cm, and 3.5 cm.

12.9 Finding the centre angle of a regular polygon

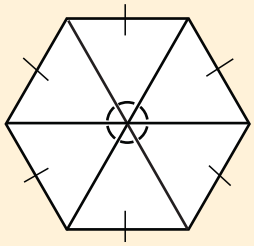
Activity 12.9

- Draw a square ABCD.
- Draw diagonals AC and BD. Mark the point O at which the two diagonals bisect each other. Is the point O the center of the square? Explain your answer.



How many angles are formed at the centre. Measure them. What is their measure?

- Discuss with your collegue how you can find the center of a regular hexagon.



- Draw lines connecting vertices through the centre. Count and measure them.
- How do the angles relate to the number of sides?
- Share the procedure with classmates and come up with a conclusion

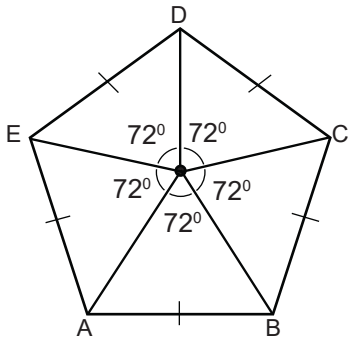
Summary

- The centre angle of a regular polygon is equal to the sum of centre angles divided by the number of centre angles.
- The centre angle sum of a polygon is 360^0.
- Number of centre angles = Number of exterior angles = Number of sides of a polygon.
- Centre angle = exterior angle of a regular polygon.

$$\text{Centre angle} = \frac{360^0}{number\ of\ centre\ angles}$$

Example 1

Find the centre angle of a regular pentagon.



Solution

Angles at a point add up to 360^0. A pentagon has 5 centre angles.

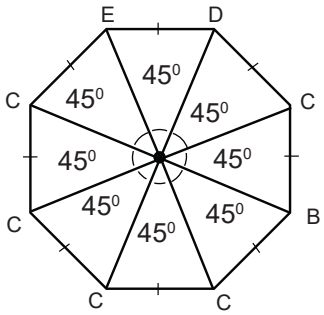
$$Each\ centre\ angle = \frac{sum\ of\ centre\ angles}{number\ of\ centre\ angles}$$

$$= \frac{^{72}\cancel{360^0}}{\cancel{5}_{1}} = 72^0$$

The centre angle of a regular pentagon is 72°

Example 2

Find the centre angle of a regular octagon.



Solution

An octagon has 8 centre angles.

Angles at a point add up to 360^0.

$$Each\ centre\ angle = \frac{sum\ of\ centre\ angles}{number\ of\ centre\ angles}$$

$$= \frac{^{45}\cancel{360^0}}{\cancel{8}_{1}} = 45^0$$

The centre angle of a regular octagon is 45 degrees.

Application activity 12.9

Find the centre angles of the regular polygons below:

(a) Equilateral triangle (b) Septagon (c) Nonagon

(d) Decagon (e) Hexagon

12.10 Constructing a regular pentagon and regular hexagon

Activity 12.10

- Draw a circle on a sheet of paper.
- Mark the centre then draw its radius.
- Measure an angle of 72^0 at the centre.
- Draw another radius connecting to the circle. Name the point B.
- Place the compass needle at point A and the pencil at B. Using the same length, place at B to make an arc at C.
- From C, make an arc at D, then from D, make an arc to make point E.
- Using a ruler, connect the points A to B, B to C, C to D, D to E and E to A.
- How many sides has the polygon? Name the polygon.
- Measure each side. What is the measurement?
- Demonstrate and present to the class your procedure and findings.
- Carry out the same steps for constructing a regular hexagon but this time use the radius distance to make 6 arcs.
- Demonstrate and make a presentation to class.

Summary

- The word pentagon means "a 5 sided figure".
- A regular pentagon has equal sides and equal centre angles of 72^0.
- The word hexagon means "a 6 sides figure".
- A regular hexagon has equal sides and equal centre angles of 60^0.
- Centre angle = exterior angle of a regular polygon.

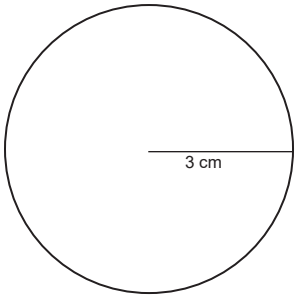
$$\text{Centre angle} = \frac{360^0}{\textit{number of centre angles}} = \frac{360^0}{\textit{number of sides}}$$

Example 1

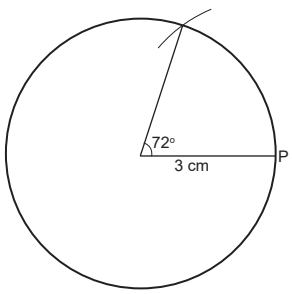
Construct a regular pentagon of radius 3 cm.

Solution

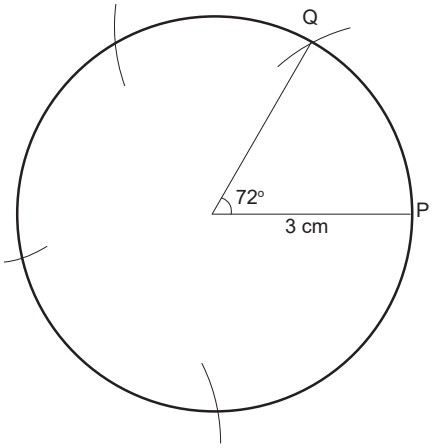
Step 1: Draw a circle of radius 3 cm.



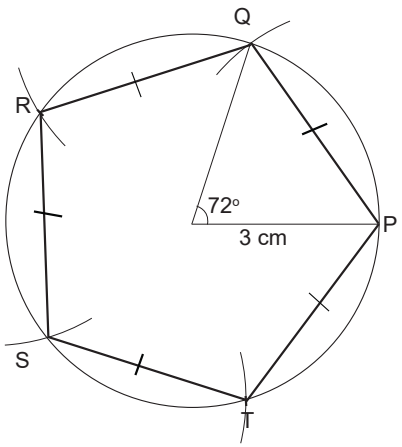
Step 2: Draw a centre angle of 72^0.



Step 3: Place the compass needle at point P and the pencil point at point Q using the length AB, make arcs of equal length along the circle.



Step 4: Using a ruler, draw straight lines connecting the arcs to form edges. Name the vertices R, S and T.

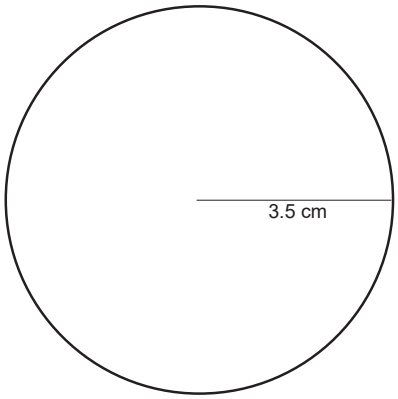


Example 2

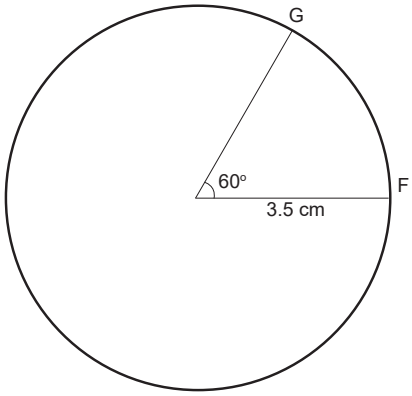
Construct a regular hexagon of radius 3.5 cm.

Solution

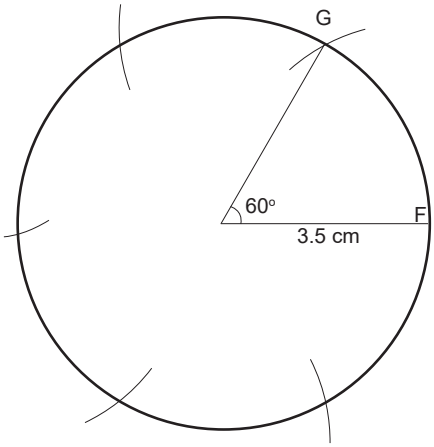
Step 1: Draw a circle of radius 3.5 cm.



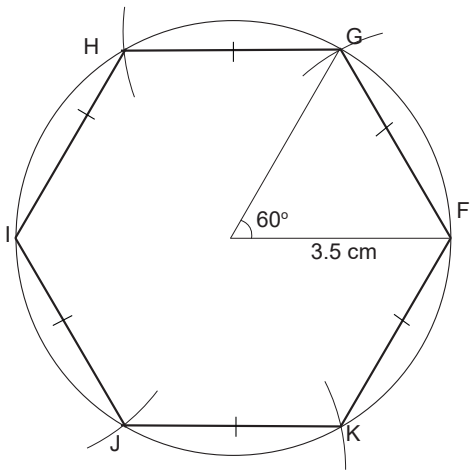
Step 2: Draw a centre angle of 60^0.



Step 3: Place the compass needle at point F and the pencil point at G. Using the length PQ, make arcs of equal length along the circle.



Step 4: Using a ruler, draw straight lines connecting the arcs to form edges. Name the vertices H, I, J, K.



Application activity 12.10

1. Construct regular pentagons with radius:
   (a) 3.5 cm (b) 4 cm (c) 4.8 cm
   (d) 5 cm (e) 5.5 cm (f) 7 cm
2. Construct regular hexagons with radii:
   (a) 3.6 cm (b) 4.2 cm (c) 4.5 cm
   (d) 5 cm (e) 5.6 cm (f) 8 cm

12.11 Constructing a regular heptagon and a regular octagon

Activity 12.11

- Draw a circle on a sheet of paper.
- Mark the centre then draw its radius.
- Using a protractor, draw an angle of 45^0 at the centre.
- Using compasses and pencil, measure the length of the arc along which the two radii from the centre connect.
- Make an arc where the 45^0 line connects the circle.
- Do not change the radius of the compass and pencil.
- Place the compass point at the previous arc and make another arc along the circle.
- Continue doing the same completely around the circle.
- Using a ruler, connect the arcs with straight lines.
- How many sides has the figure?
- Using a protractor, measure each interior angle. What do you notice?
- Write your observations and present to the class.

Summary

- The word heptagon means "a 7 sided figure".
- A regular heptangon has 7 equal sides and equal centre angles of approximately 51^0.
- The word octagon means "an 8 sides figure".
- A regular octagon has 8 equal sides and equal centre angles of 45^0.
- To construct regular septagon and octagon, first find the centre angle.
- Draw arcs at equal length along the circle, then join them with straight lines.
- Centre angle = exterior of a regular polygon.

$$\text{Centre angle} = \frac{360^0}{\text{number of centre angles}} = \frac{360^0}{\text{number of sides}}$$

Example 1

Construct a regular septagon DEFGHIJ using radius 3.2 cm.

Solution

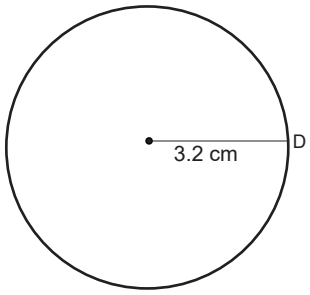
Step 1: Find the centre angle. A septagon has 7 equal sides. It has 7 equal centre angles.

$$\text{Each centre angle} = \frac{360^0}{\text{number of centre angles}}$$

$$= \frac{^{51.436}\cancel{360^0}}{^{1}\cancel{7}} = 51.436^0$$

Round it off to 51^0.

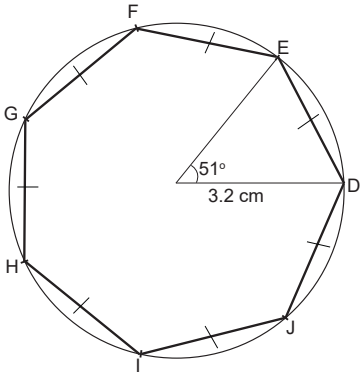
Step 2: Draw a circle of radius 3.2 cm. Name point D at the circle.



Step 3: Draw an angle of 51^0 at the centre. Name point E along the 51^0 line on the circle.

Step 4: Measure the length of the arc between the lines that form 51^0 along the circle. Make arc at Without adjusting the radius between the compass point and pencil, make other arcs along the circle.

Step 5: Draw straight lines connecting the arcs along the circle.



Step 6: Label the remaining vertices FGHIJ and the sides with equal signs. The figure is a regular septagon.

Example 2

Construct a regular octagon ABCDEFGH using radius 3 cm.

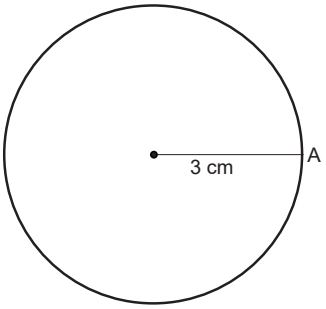
Solution

Step 1: Find the centre angle. An octagon has 8 centre angles.

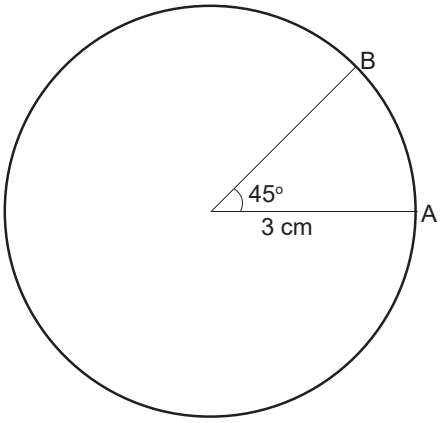
$$\text{Each centre angle} = \frac{360^0}{\text{number of centre angles}}$$

$$= \frac{^{45}\cancel{360^0}}{^{1}\cancel{8}} = 45^0$$

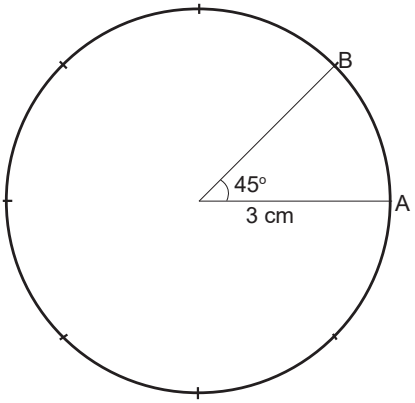
Step 2: Draw a circle of radius 3 cm. Name point A at the circle.



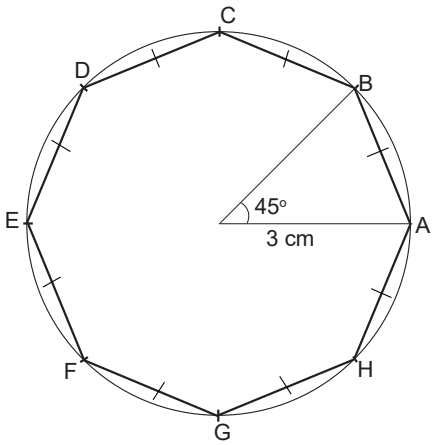
Step 3: Draw an angle of 45^0 at the centre. Name point B on the circle.



Step 4: Measure the length between radius and 45^0 line. Make similar arcs along the circle without altering the length.



Step 5: Draw straight lines connecting the arcs along the circle. Label all the vertices with letters and equal marks.



The figure is an octagon.

Application activity 12.11

Using a ruler and compasses, construct the following regular polygons:

(a) Septagon of radius 3 cm.

(b) Septagon of radius 3.6 cm.

(c) Septagon of radius 4 cm.

(d) Octagon of radius 3.8 cm.

12.12 Constructing a regular nonagon and decagon

Activity 12.12

- On slips of paper, draw a circle of any radius.
- Measure a centre angle of 40^o.
- Using compasses and a pencil, measure the length of arc between the radius and 40^o line. Make an arc.

- Without adjusting the compass, draw similar arcs along the circle.
- Join the arcs with straight lines.
- How many sides does the figure have?
- What do you observe about its sides?
- Measure the interior angles. What do you realise?
- Name the figure.

Summary

- A regular nonagon has equal sides and equal centre angles of 40^0.
- The word decagon means "a 10 sides figure".
- A regular decagon has equal sides and equal centre angles of 36^0.
- To construct regular nonagon and decagon, first find the centre angle.
- Draw arcs at equal length along the circle, then join them with straight lines.
- Centre angle = exterior of a regular polygon.

$$\text{Centre angle} = \frac{360^0}{\textit{number of centre angle}}$$

Example 1

Using compasses and ruler, construct a regular nonagon PQRSTUVWX of radius 4 cm.

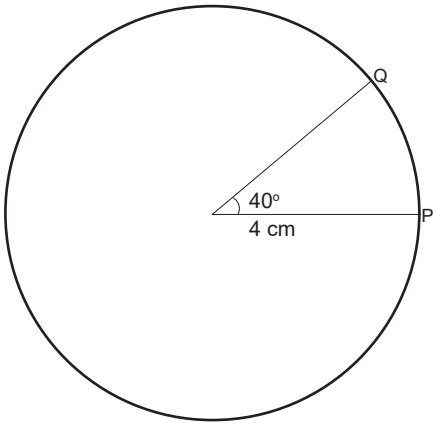
Solution

Step 1: Draw a circle of radius 4 cm. Name point P on the circle.

Step 2: Find the centre angle.

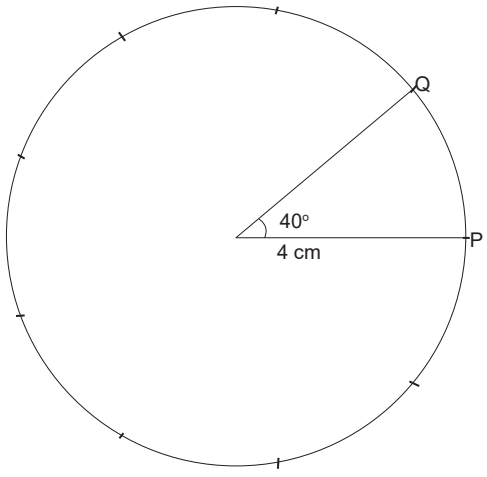
$$\textit{Centre angle} = \frac{360^0}{9} = 40^0$$

Step 3: Draw a centre angle of 40^0 in the circle.

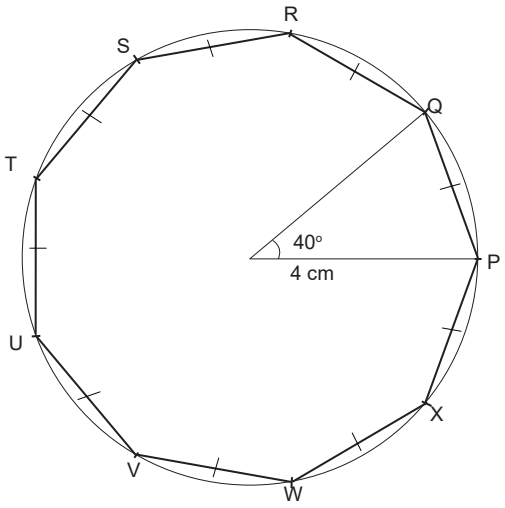


Step 4: Measure the distance between P and Q using compasses and a pencil. Draw an arc at Q.

Step 5: Without adjusting the compass, make other arcs along the circle.



Step 6: Using a ruler, draw straight lines connecting the arcs.



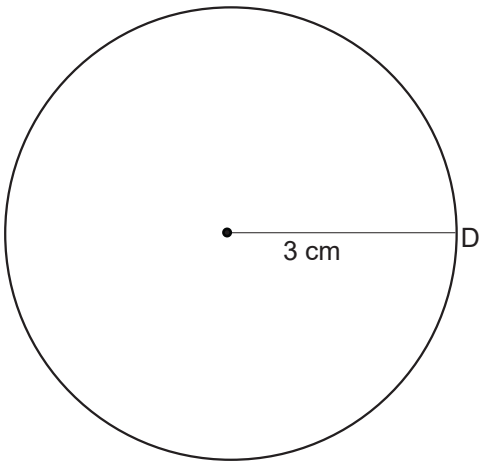
Step 7: Complete labelling the vertices. Mark sides with equal symbols. The regular polygon is a nonagon PQRSTUVWX.

Example 2

Construct a regular decagon of radius 3 cm.

Solution

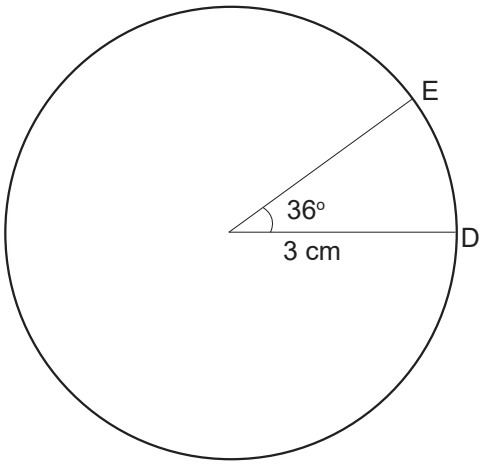
Step 1: Draw a circle of radius 3 cm.



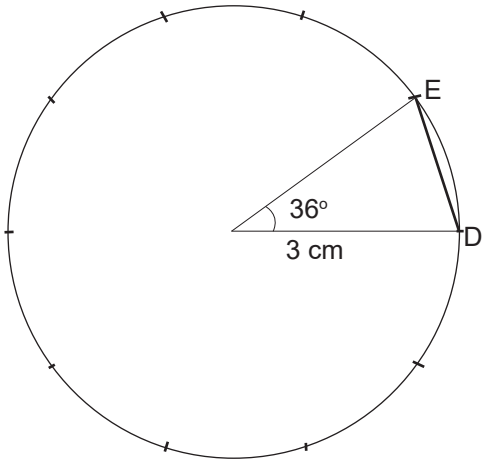
Step 2: Find the centre angle of a decagon.

$$\text{Centre angle} = \frac{360^0}{10} = 36^0$$

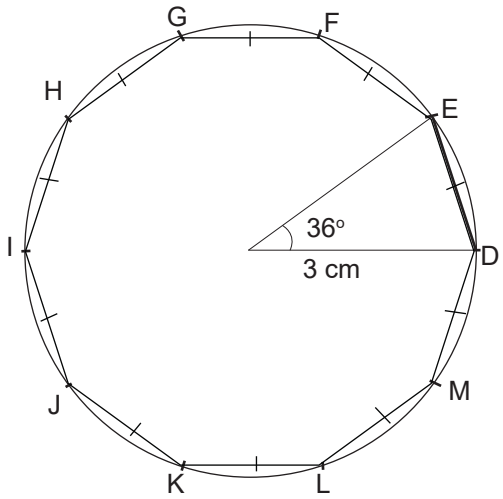
Draw an angle of 36° in the circle.



Step 3: Measure the length of arc DE. Using compasses and a pencil, make arcs along the circle.



Step 4: Join the arcs with straight lines.



Step 5: Complete labelling other vertices DEFGHIJKLM. Name the polygon. The polygon is a regular decagon.

Application activity 12.12

1. Construct regular nonagons with radii:
   (a) 3.2 cm (b) 3.5 cm (c) 4 cm (d) 4.3 cm
2. Construct regular decagons with radii:
   (a) 2.8 cm (b) 3.4 cm (c) 4.5 cm (d) 6 cm

12.13 Project for designing nets of cuboids, cubes and prisms

Activity 12.13

- Get an empty box of chalk and unfold it carefully.
- Display the flat shapes that were folded to form it.
- Draw the net of the flat shapes displayed.
- Present your working out to the rest of the class.

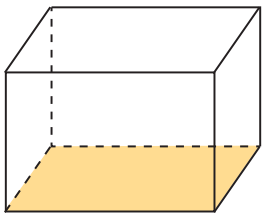
Summary

A net is a flat shape that folds up to make a prism.

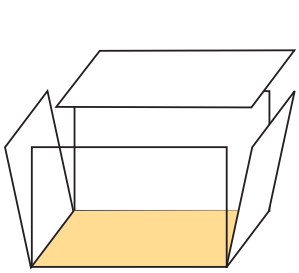
A prism is a three-dimensional figure with ends that are identical polygons.

Example 1

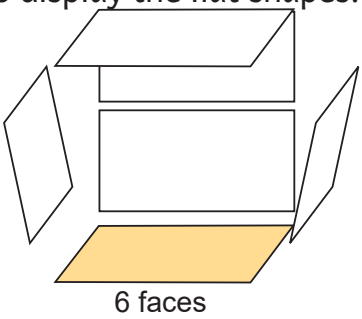
Given the cuboid, display the flat shapes and draw its net.



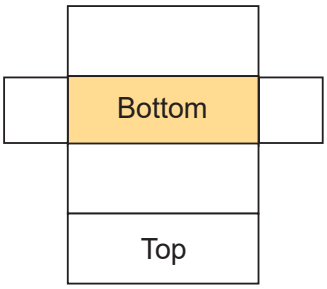
Solution: First unfold it to display the flat shapes.



Open the box

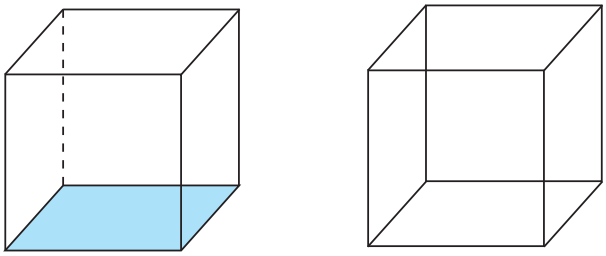


Next, cut the six faces



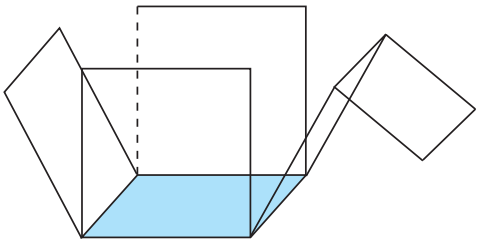
Finally, draw the net.

Example 2: Make a net for the cube.

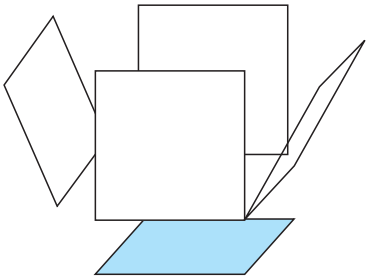


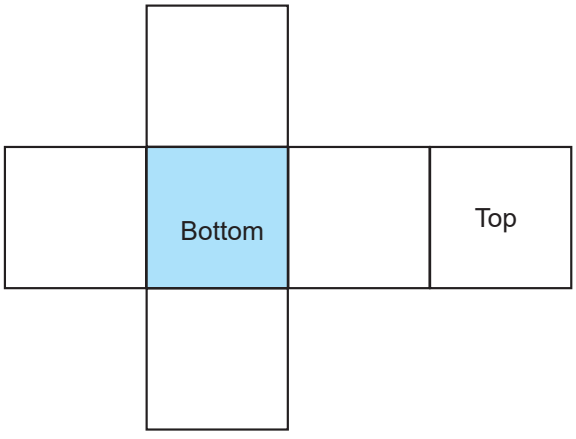
Solution: First unfold the cube to display the flat shapes.

Open the cube box.



Cut out the faces.



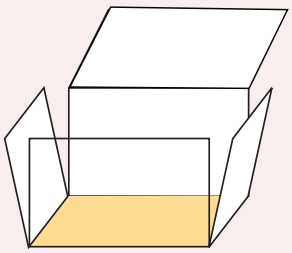


Finally, draw the net.

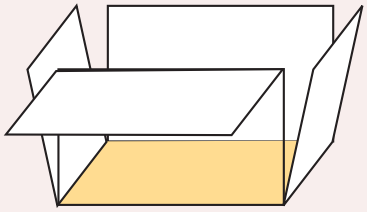
Application activity 12.13

1. Design nets for the following:

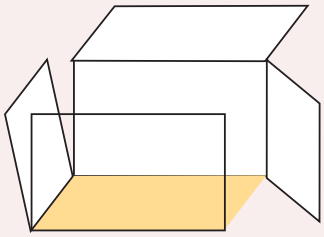
(a)



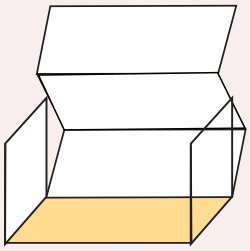
(b)



(c)

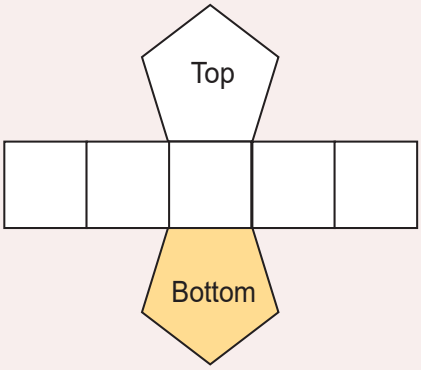


(d)

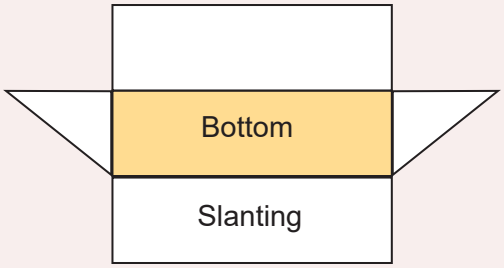


2. From the nets below, draw the prism.

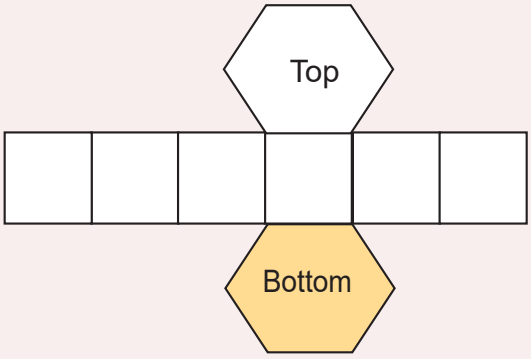
(a) Pentagonal prism



(b) Triangular prism



(c) Hexagonal prism



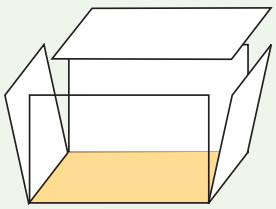
12.14. End of unit assessment

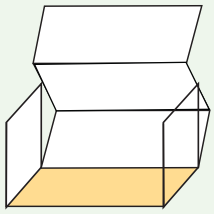
A. Assessing knowledge and understanding

1. Definitions and Properties:
   a) What are the properties of a square?
   b) How many sides does a regular pentagon have? What is the measure of each interior angle?
   c) What is the difference between a rectangle and a cuboid?
2. Calculations:
   a) Find the centre angle of a regular nonagon.
   b) A regular hexagon has a side length of 4 cm. What is its perimeter ?
3. Identifying Shapes:
   a) Which of the following shapes are polygons: triangle, square and cylinder?
   b) Name two real-life objects that have the shape of a prism.

B. Assessing skills

4. Using a protractor and ruler, draw an equilateral triangle PQR of sides 5.5 cm
5. Draw a pentagon of sides 4.5 cm. Label it ABCDE. Use ruler and a protractor.
6. Using ruler and compasses only, construct the following polygons:
   a) A square with sides 5.6 cm
   b) A rectangle with length 6 cm and width 3 cm.
   c) Triangle ABC with AB = 5.5 cm, angle A = 80^0 and angle B = 50^0.
7. Copy and shade one pair of the bottom and top faces of the prisms below.

(a) 

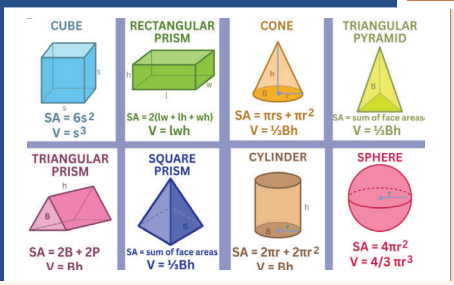
(b) 

8. Using ruler and compasses, construct the regular polygons below:
   (a) Hexagon ABCDEF with side 3.5 cm.
   (b) Octagon PQRSTUVW with side 2.8 cm.

C. Assessing attitudes and values:

9. Why is it important for architects to understand nets when designing buildings?
10. Answer to the following question:
    (a) Why is it important to use precise measurements when constructing polygons?
    (b) How does understanding geometric constructions help in designing objects like furniture or packaging?
11. Read and answer:
    (a) If a shape has all sides equal but not all angles equal, is it a regular polygon? Explain.
    (b) Can a net of a cube be designed in more than one way? Justify your answer.

UNIT 13



AREA BOUNDED BY A CIRCLE, SURFACE AREA AND VOLUME OF SOME SOLIDS

Key Unit Competence: You will be able to calculate the area enclosed by a circle, the surface area and volume of cylinder, cone, sphere, prism and pyramid.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- State the formula for finding the area bounded by a circle and explain how it can be derived from the circumference of a circle.
- Explain the surface area of a cuboid as the area of its net.
- State formula for the volume of a solid

Skills:

- Calculate the area bounded by a circle.
- Use the nets of a solid to determine its surface area.
- Calculate the volume of a solid.
- Select appropriate units when calculating are and volume.

Attitudes and values:

Appreciate the difference between area, surface area and volume and the importance of using the correct units.

13.0 Introduction

Surface area is the total area covering the outside of a shape. For flat shapes like circles, we call this the area, while for solid objects like boxes or balls, it is the sum of all their faces. We use surface area in real life when wrapping gifts, painting walls, or designing packaging; it helps us to know how much material is needed to cover objects.

Volume, on the other hand, measures how much space a solid object takes up. It tells us how much a container can hold, like water in a bottle or toys in a box. Builders use volume to calculate materials, cooks measure ingredients, and even shipping companies rely on it to pack goods efficiently. Understanding these concepts helps us to solve everyday problems such as the size of the sheet of paper to wrap/cove a gift or the size for the iron sheet to make a metallic box.

Introductory Activity

When you observe the area of the rectangle, square, triangle or circle, you can find that it is made up small squares.

a) Take a grided paper from the exercises book: It is made of small squares. Try to use a ruler to measure the side of each small square, then find its area.

b) Try to count the total number of small squares for the entire side of the sheet of paper. Can you determine the total area of the sheet of paper using the number you got?

c) Now use a ruler to measure the length and the width of the sheet of paper. Can you use them to find the area of the sheet of paper?

d) Compare your findings in b) and c) and then, discuss how you can find the area of a plane surface.

e) Discuss if you can explain how to find the area of any shape observed in the nature.

13.1 Estimating the area bounded by a circle using a squared paper

Activity 13.1

- Draw a circle of a radius of 3 squares on gridded paper for which each unit square has 1 cm^2
- Estimate the area enclosed by counting the small squares.
- Add two parts of squares which do not form complete squares to form 1 small square.
- Practice more with a radius of 2 squares, then 3 squares.
- Tabulate the results in the table below:

	Radius (r)	r^2	$3.14 \times r^2$	(Total number of unit squares) x cm^2	Are they the same?
Circle 1					
Circle 2					
Circle 3					

- What do you observe?
- Discuss the relationship in the tabulated values.
- Share the procedure and outcomes with other groups.

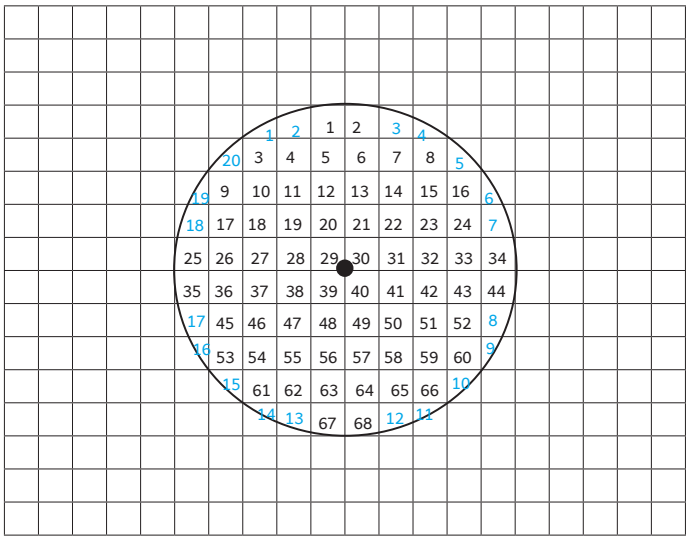
Summary

- Area of a circle is the region bounded inside it.
- Area is equal to the number of square units inside its boundary.
- Area divided by r^2 is equal to $Pi(\pi)$
- $Area = \pi \times r \times r = \pi \times r^2$
- When the unit square has the area = Scm^2, the area of a circle is equal to the product of the total number of unit squares N by S cm^2.

  Area = N x S cm^2

Example

Using squared paper, find the area of a circle of radius of 5 squares



There are 68 whole squares + 20 portions of a squares.

Area = 68 squares + 10 squares.

Area = 78 square units

- Area of a circle is the region bounded inside it.
- Area is equal to the number of square units inside its boundary. This means the area of the circle = 78 square units. If one unit square = 1cm^2, The area = 78cm^2
- This area is equal to the product of the radius squared by a constant number Pi.

This means the area of a circle $Area = \pi \times r \times r = \pi \times r^2$

Application activity 13.1

1. Using gridded paper, find the area of a circle whose radius is:

   (a) 3 unit squares (b) 4 unit squares (c) 6unit squares

   (d) 7 unit squares (e) 8 unit squares (f) 10 unit squares

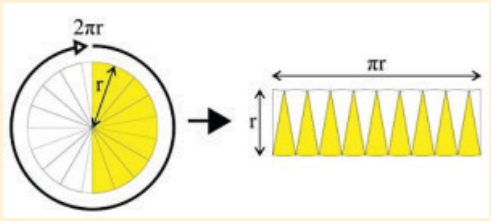
2. Using the results in number 1 above, complete the table below:

Radius (r)		r^2	Area (A)
(a)			
(b)			
(c)			

13.2 Exploring the area bounded by a circle using the concept of circumference and radius

Activity 13.2

Use circular sheets divided into 4, 6, 8, 12 and 16 equal parts (sectors).



- Let us use a circle on a paper and divide it into 16 segments.
- Now cut it out and cut it into 16 segments.
- Arrange the segments to approximate a parallelogram as shown.

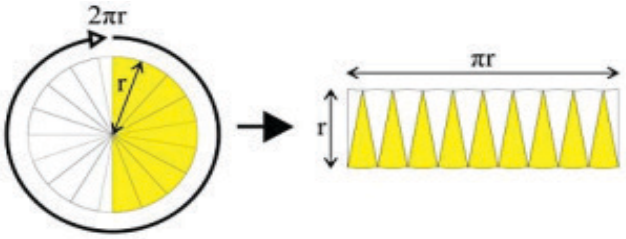
- Link its dimensions to the circumference and radius. What is the length of the parallelogram, what is its height?
- Use the length and height to find area A of your parallelogram. A =
- Now, refer to your findings and establish the formula for calculating area of the initial circle.

Area of the circle = ...

Summary

Area of a circle:

Linking the dimensions of the parallelogram formed and those of the initial circle, we have the following:



Length equals to the half of the circumference of the circle of radius r.

This is $L = \frac{C}{2} = \frac{2\pi \times r}{2} = \pi \times r$

The height of the parallelogram is equal to the radius r of the circle. This means $H = r$

Therefore, the area of the circle is equal to the area of the parallelogram it makes. This is

$A = L \times H = (\pi \times r) \times r = \pi \times r^2$

- The area A of a circle with radius R is equal to $A = \pi \times r^2$ where π is the constant and $\pi \approx 3.14$.
- Area is calculated in square units. For example, mm^2, cm^2, dm^2, m^2, dam^2, hm^2, km^2.
- Area is the region or space bounded or enclosed by a boundary.

Example 1

Calculate the area of a circle whose radius 5cm.

Solution

Radius = 5 cm, Pi=3.14

Area of a circle = $\frac{1}{2}(2\pi r)r$

= πr^2 = 3.14 x 5cm x 5cm=78.5 cm^2

Example 2

A saucepan has a circular bottom. Its radius is 0.7m. What is its area in cm^2 if pi=$\frac{22}{7}$

Solution

Radius = 0.7m,

area $= \frac{22}{7} \times 0.7m \times 0.7m = 22 \times 0.1m \times 7m^2 = 1.54m^2$

But 1 m = 100 cm

1 m^2 = 10,000 cm^2

1.54 m^2 = 1.54 x 10,000 cm^2

= 15,400 cm^2

We can calculate the area of a circle using its diameter D.

As $r = \frac{D}{2}$, $Area = \pi \times \frac{D}{2} \times \frac{D}{2} = \pi \times \frac{D^4}{4}$.

Example 1

Find the area of a circle with a diameter of 28cm. pi=$\frac{22}{7}$

Solution

Find the radius:

Radius = $\frac{\text{diameter}}{2} = \frac{28cm}{2}$ 28 = 14cm

Area of a circle = πr^2

= ($\frac{22}{7}$) x 14 cm x 14 cm

= 2,156 cm^2

Example 2

A saucepan has a circular bottom. Its diameter is 30 cm. What is its area in cm^2 if π = 3.14?

Solution

Find the radius:

Radius = $\frac{\text{diameter}}{2} = \frac{30cm}{2}$ = 15cm

radius = 15 cm, π = 3.14

Area of a circle = πr^2

= 3.14 x 15 cm x 15 cm

= 3.14 x 225 cm^2

= 706.5 cm^2

We can calculate the area of a circle if its circumference is given.

Given that the Circumference $C = 2\pi \times r$, the radius $r = \frac{C}{2\pi}$.

Therefore, The area A of a circle $A = \pi \times r^2 = \pi \times \frac{C}{2\pi} \times \frac{C}{2\pi} = \frac{C \times C}{4\pi} = \frac{C^2}{4\pi}$

Example 1

The circumference of a circular table is 132 cm. What is its area?

Solution

Circumference = $2\pi r$, $\pi = \frac{22}{7}$

$132 = 2 \times \frac{22}{7} \times r$

$132 = \frac{44}{7} \times r$

$r = \frac{132 \times 7}{44} cm = 21cm$

Area of a circle = πr^2

r = 21cm, $\pi = \frac{22}{7}$

Area of a circle = πr^2 =

$\pi r^2 = \frac{22}{7} \times 21cm \times 21cm = 1386\,cm^2$

Example 2

The circumference of a circular garden is 125.6 m. Calculate its area.

Solution

Circumference = $2\pi r$

125.6 = 2 × 3.14 x r

125.6 = 6.28r

Divide both sides by 6.28

$r = \frac{125.6}{6.28} cm = 20cm$

r = 20cm

Area of a circle = πr^2

r = 20cm, π = 3.14

Area of a circle = π r²

= π x r x r

= 3.14 x 20 m x 20 m

= 314 x 4 m²

= 1256 m²

The area of the circular garden is 1256 m².

We can find the radius using area of a circle

Area = πr^2 ; $r^2 = \frac{area}{\pi}$

When you have: $r^2 = r \times r$, you can find the value of r

Example: Area of a circle is 113.04 m^2 . What is its radius? Use $\pi = 3.14$.

Solution: Area = πr^2

113.14= 3.14 x r^2 , r^2 =113.14÷3.14 = 36

$r^2 = r \times r = 36$

$r = 6$ m

The radius of the circle is 6m.

Application activity 13.2

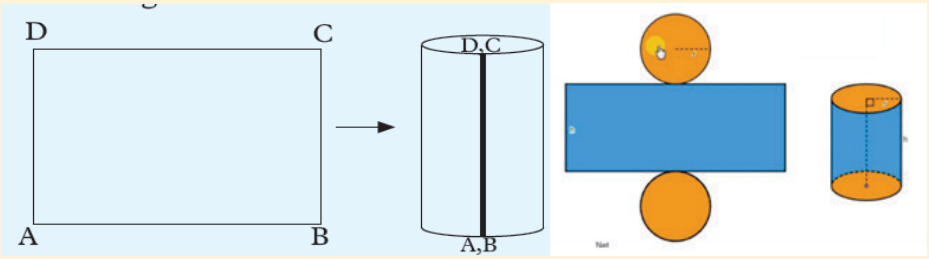
1. Find the area of the circle with the following radius for $\pi \approx \frac{22}{7}$:
   (a) 28 cm (b) 14 cm (c) 7 cm (d) 35 cm
2. A circular top of a dining table has a radius 4.5 cm. Find its area. Use $\pi \approx \frac{22}{7}$.
3. Find the area of a circle whose diameter is 20 cm.
4. A circular garden has a diameter of 40 m.Find its area.
5. The circumference of a circle is 88 cm. Calculate its area.
6. The circumference of a circular field is 628 cm. Calculate its area.
7. A circular plot of land has a circumference of 314 m. Calculate its area.
8. Find the radius of a circle with area of 78.5 cm^2 . Take pi =3.14.
9. What is the radius of a circle of area of 28.16 cm^2 ?
10. The area of a circular table is 50.24 cm^2 . Find the radius.

13.3 Surface area of cylinder

Activity 13.3

– Obtain a rectangular manila piece of paper. Label it A, B, C, D such that AB= 10cm and DC=10cm.

- Find the area of the rectangle ABCD. Without folding, join edges AD onto CB using glue as shown in the following figure:



– What is the name of the distance round DC and AB?

- Cut a paper that fits round the circumference of the cylinder you made.
- Trim any excess paper with a pair of scissors so that the circular cut out does not overlap.
- Calculate the area of the paper cut out. How did you find radius of these end faces?
- What is the total surface area of the can?
- Obtain a can and construct its net.
- Make appropriate measurements and record them on the net.
- Find the surface area of the can.

Summary

The net of a cylinder is made up of **one rectangle** whose length is equal to the height of the cylinder and the width is equal to the circumference of its circular cross-section, and the two ends of a closed cylinder faces which are **circles**.

The surface area of a closed cylinder is composed of the sum of the curved surface and the two end faces.

Surface area of **Open cylinder** equals the surface area of one circular base plus surface area of 1 rectangular face $= \pi \times r^2 + (2\pi \times r) \times h$

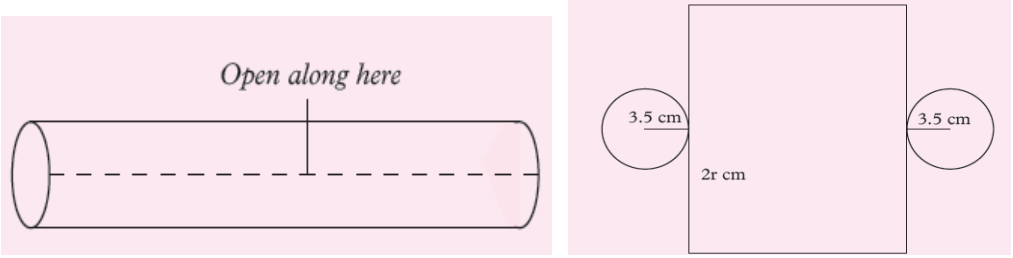
Area of a closed cylinder equals the surface area of two circular base plus surface area of one rectangular face $= 2(\pi \times r^2) + (2\pi \times r) \times h$

Example

1. A closed cylinder whose height is 18 cm has a radius of 3.5 cm. Draw the net of the cylinder and use it to find the total surface area of the cylinder.

Solution

The diagram of the cylinder and its nets



Radius= 3.5cm, height = 18cm. **π=22/7**

The length of the rectangle = Circumference of one the end face (circle) = 2πr = 22 cm.

The net consists of two end circles, and a rectangle measuring 22 cm by 18 cm.

Total surface area $2 \times \pi r^2 + (2\pi r) \times h$

$= 2 \times \pi \times (3.5\ cm)^2 + 22\ cm \times 18\ cm$

$= (76.98 + 396)\ cm^2 = 472.98\ cm^2$

2. A very thin sheet of metal is used to make a cylinder of radius 5 cm and height 14 cm. Using π = 3.14, find the total area of the sheet that is needed to make:
   a) a closed cylinder
   b) a cylinder that is open on one end.

Solution

a) Radius of the circular face of the cylinder = 5 cm

Area of a circular face = $\pi r^2 = (3.142 \times 5 \times 5)\ cm^2$

Area of the two circular end faces = $2 \times \pi r^2 = (2 \times 3.142 \times 5 \times 5)\ cm^2$

Recall that when a cylinder is opened up to form its net, the curved surface becomes a rectangle of length 2πr (i.e. the circumference of the cylinder) and width h (the height of the cylinder).

Thus, area of curved surface = 2πr × h = (2 × 3.142 × 5 × 14) cm^2

Now, total surface area of the metal sheet

= [(2 × 3.142 × 5 × 5) + (2 × 3.142 × 5 × 14)] cm^2

= 2 × 3.142 × 5(5 + 14) cm^2 = 596.98 cm^2

(b) Surface area of open cylinder

$\pi r^2 + 2\pi rh$ = (3.142 × 5 × 5) + (2 × 3.142 × 5× 14)

= 3.142 × 5(5 + 2 × 14) cm^2 = 3.142 × 5 × 33 cm^2 = 518.43 cm^2 .

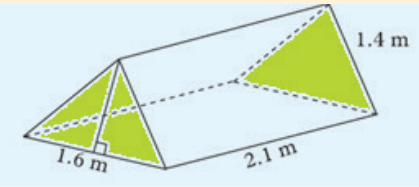
Application activity 13.3

1. A cylinder has a height of **12 cm and a radius of 4 cm**.
   a) Draw the **net of the cylinder and label:**
      - The **rectangle (length and width).**
      - The **two circular faces (radius).**
   b) Calculate:
      - The **circumference of the circular face (use π = 3.14).**
      - The **area of the rectangle in the net.**
2. A **closed cylindrical has radius = 7 cm** and height = **10 cm (Use π = 22/7).**
   a) Find the **area of the two circular faces**.
   b) Find the **area of the curved surface (rectangle).**
   c) What is the **total surface area of the can?**
3. A school wants to make a **closed metal drum (radius = 5 cm**, height = **20 cm**) and an **open-top pencil holder (same dimensions).**
   a) Calculate the total metal needed for the closed drum.
   b) Calculate the metal needed for the open-top holder. (Use π = 3.14).

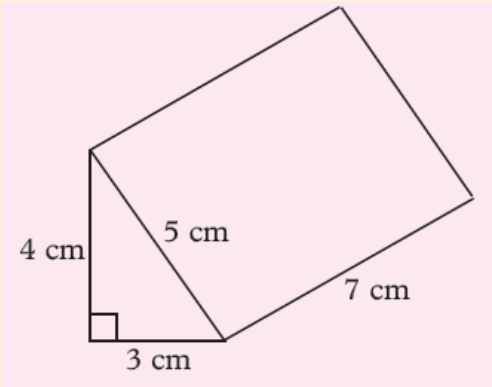
13.4 Surface area of prism

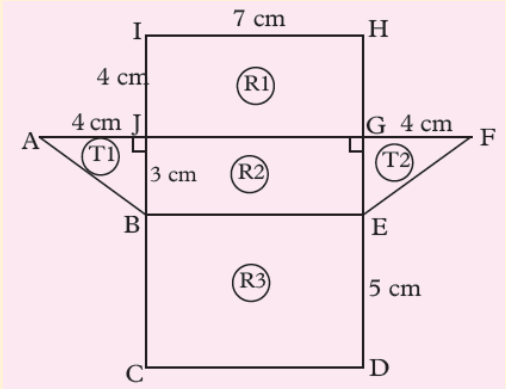
Activity 13.4

1. Tell your partner what a prism is.



2. The figure shows the prism and its nets. Use it to find the surface area of the prism.





Summary

A prism is a solid with a uniform cross-section. The cross-section may take the shape of any polygon such as triangle, square, rectangle, hexagon etc.

The faces of a prism other than the cross-section (or the bases) are called **lateral** faces of the prism. The edges joining pairs of lateral faces are called **lateral** edges

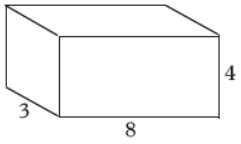
- A prism has two parallel bases.
- A prism has as many lateral faces as the number of sides of its base they are rectangles or parallelograms.
- The bases of a prism are identical.
- The lateral edges of a prism are equal and parallel.
- The lateral faces of a prism are perpendicular to the bases.
- If the base of a prism has **n** sides, then the prism has **n** lateral faces and since any prism has two bases, the total number of faces is 2 + n lateral faces.

The total number of faces = (n + 2) faces where **n** is the number of lateral faces.

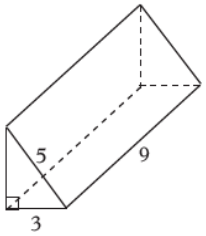
- Total surface area of a prism = 2 (area of one base) + areas of the n lateral faces

Example: All the following figures represent prism. To name a prism, we use the base. A squared prism, a rectangular prism is a cuboid. A triangular prism is a pentahedron, A circular prism is a cylinder.Name each prism and use the appropriate nets in each case to calculate the total surface area of the solids (All measurements are in cm.)

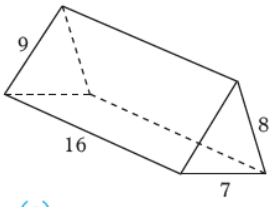
(a)



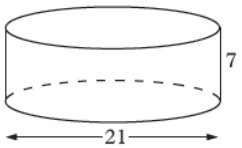
(b)



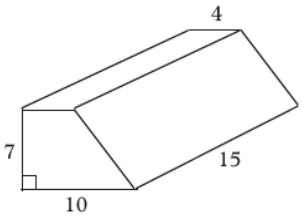
(c)



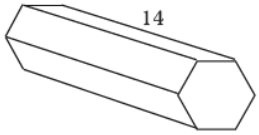
(d)



(e)



(f)



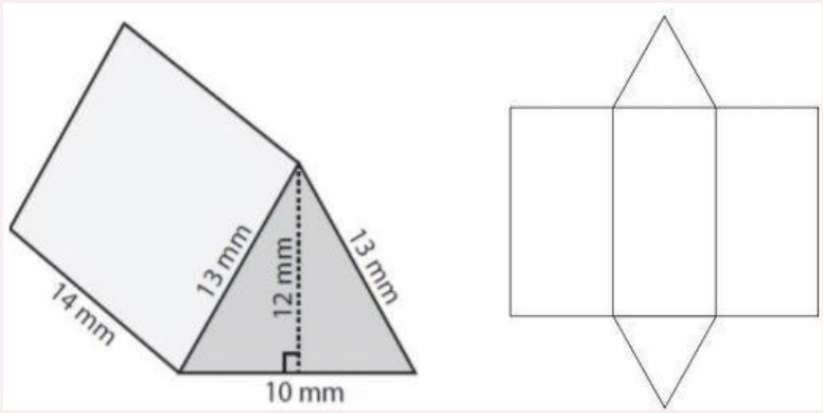
End-face is a regular hexagon of side 6 cm

Answers: (a) A= 136 cm^2 (b) A= 120 cm^2 (c) A=437.67 cm^2

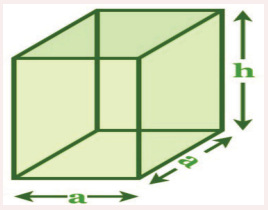
(d) A= 11154.685 cm^2 (e) A= 551.29 cm^2 (f) A= 519.589 cm^2

Application activity 13.4

1. Find the total surface area of the prism whose diagram is here below:



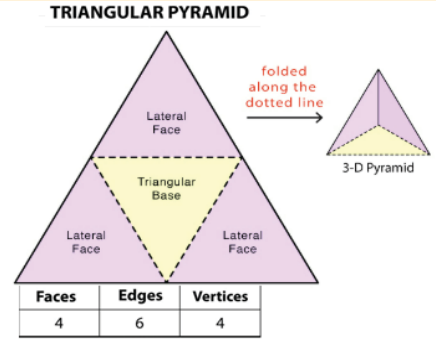
2. The side of the base of square prism is 5cm. Find the surface area of the prism if its height is 8cm.



13.5. Surface area of pyramid

Activity 13.5

Observe the following pyramids:



TRIANGULAR PYRAMID		
Faces	Edges	Vertices
4	6	4

SQUARE PYRAMID		
Shape	Nets	
Faces	Edges	Vertices
5	8	5

RECTANGULAR-BASED PYRAMID		
Shape	Nets	
Faces	Edges	Vertices
5	8	5

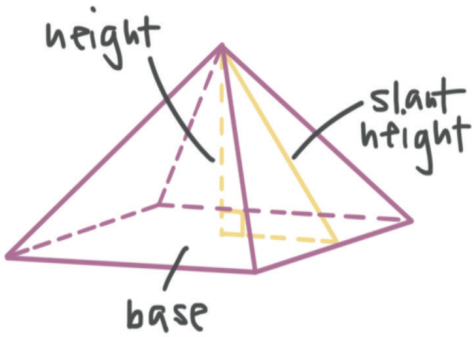
Make the same nets and discuss how to find the surface area in the following ways:

- What shapes do faces of the pyramid have?
- Record the measurements on the net.
- How many identical faces does the pyramid have?
- Calculate the area of one of the triangular faces.
- Find the total area of the net.
- What is the total surface area of the pyramid?

Summary

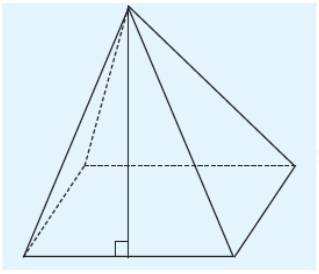
- The altitude of a slant face of a pyramid is called a slant height of the pyramid. It is perpendicular to an edge of the base.

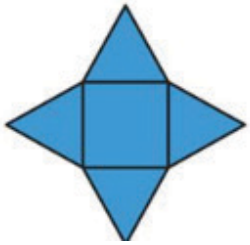
The **slant height is also called the lateral height**, and is often represented in formulas with the variable l.



- **Surface area of a pyramid = total area of the slant faces + the area of base**.
- If the vertex of a pyramid is vertically above the centre of the base, the pyramid is called a right pyramid.

Example: A pyramid with a square base of sides 5 cm. The slant height of the pyramid is 6 cm. Construct an accurate net of the pyramid.





Solution: **Slant faces are triangles.**

Area of one slant face = $\frac{1}{2} \times 6cm \times 5cm$

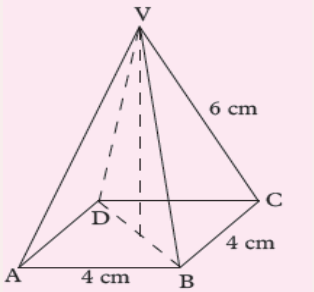
Area of 4 slant faces = $4 \times \frac{1}{2} \times 6cm \times 5cm$

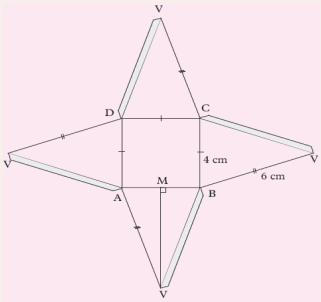
The base is a square, Area of base = (5 cm × 5cm).

Total surface area = $4 \times \frac{1}{2} \times 6cm \times 5cm$ + (5 cm × 5cm) = $85\,cm^2$

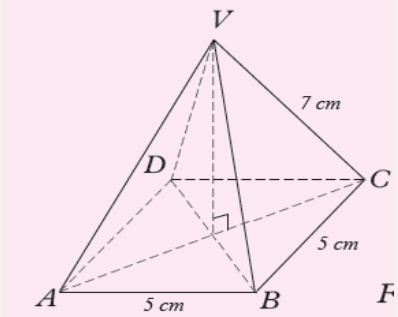
Application activity 13.5

1. The base of a right pyramid is a square of sides 4 cm. The slant edges are all 6 cm long.
   a) Draw and label a sketch of the solid.
   b) Draw a net of the pyramid.





2. Find the surface area of the right pyramid shown here below if the slant height is 6.5cm.

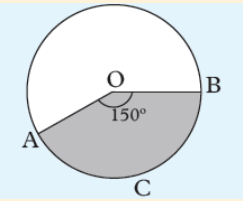


13.6 Surface area of a Cone

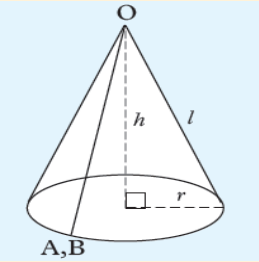
Activity 13.6

Carry out the following activities:

1. Draw a circle, radius r (say r = 10 cm). At the centre O of the circle, measure an angle AOB (e.g. 150°) and use it to form a sector (shaded part). Cut out the sector.



2. Fold the sector so that OA and OB coincide. This forms the curved surface of a cone.



The cone obtained is a **right circular cone** because it has a circular base, and the vertex is vertically above the centre of the base. The word 'right' here means 'perpendicular to the base'.

3. What fraction of the circumference is arc ACB. Calculate the length of the arc.
4. Considering the angle $AOB = \theta$, and the radius l of the circle, how could you find the area of circle sector AOB?

Summary

- The surface area of a cone is composed of the curved surface and the circular base.
- Area of the curved surface of a cone looks like the area of the triangle of base equal to the half of the circumference of the circle of radius r and height equal to the slant height l similar to the radius of initial circle of the initial circle.

This is $\pi r \times l$

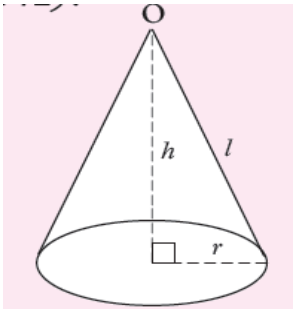
The area of the circular base= πr^2.

Hence, total surface area of a closed cone= $\pi r \times l + \pi r^2$

The length of the arc ACB = $\frac{\theta}{360} \times 2\pi l$.

Example: Find the surface area of a cone whose slant height and radius are 5 cm and 3 cm respectively. (use π= 3.14)

Solution: Using l = 5 cm, and r = 3 cm



Area of curved surface = πrl = 3.142 × 3 × 5 cm^2 = 47.13 cm^2

Area of circular base = πr^2 = 3.142 × 9 cm^2 = 28.278 cm^2

Hence, total surface area= πr^2 + πrl

= (28.278 + 47.13) = 75.408 cm^2

= 75.41 cm^2 (2 d.p)

Application activity 13.6

1. A solid cone has a radius of 8 cm and a slant height of 17cm. Calculate the total surface area of the cone.
2. The area of the curved surface area of a cone is 32.64 cm^2. If the radius of the cone is 3.7 cm, find the slant height and the total surface area.
3. Find the surface area of the given cone if its slant height is 9 cm; the perimeter of base is 12cm.

13.7 Surface area of a Sphere

Activity 13.7

1. Can we cover/wrap a Ball?
   - Take a small ball (e.g., a ping pong ball).
   - Cut square pieces of paper (1 cm × 1 cm).
   - Try covering the ball with the squares (they'll overlap because the sphere is curved!).
   - Count how many squares you used—this estimates the surface area A
   - Given that the Radius (r) of a ping pong ball is about 2 cm, compare your result A with $4\pi r^2$ where r =2cm. Are they the same?
2. (a) Obtain a well-rounded orange or any other fruit whose shape resembles that of a ball.

   (b) Carefully, cut it into halves.

   (c) Using a string or a ruler, measure the diameters of each of the two halves, ensure that they are identical.

   (d) Peel out each of the two halves carefully.

   (e) Find circular sheets of paper having the same diameter obtained in c) above. Try to use those sheets of papers to cover/wrap the orange.

   i) How many whole circles did you use to cover the orange?

   ii) What is the surface area of each circle?

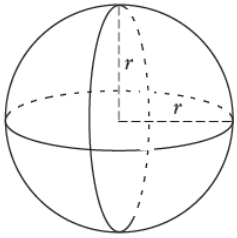
   iii) What is the total area of the four circles?

   iv) Compare the area of the four circles with the area of the sphere.

Summary

A sphere is a perfectly round shape where every point on its surface is the same distance from its center.

The sphere has no Edges or Corners, it is Smooth all around, like a bubble or ball.



From this activity, you should have observed that:

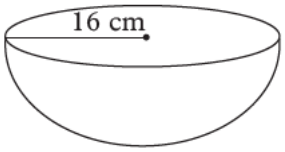
- You needed approximately **four circles** to cover/wrap the whole sphere.
- The surface area of the sphere is equal to 4 times the area of one circle i.e.

Area of one circle = πr^2 (r being the radius of the sphere).

Area of 4 circles = 4 × πr^2 = $4\pi r^2$ (since 4 circles are needed).

Surface area of a sphere = $4\pi r^2$ square units.

Half of a sphere is known as a **hemisphere**.



A hemisphere has two faces, the curved surface and the circular face. Its surface area is given by

Surface area of hemisphere:

= half the area of the sphere + area of flat surface

$$= \frac{1}{2} \times 4\pi r^2 + \pi r^2 = 3\pi r^2$$

Examples:

1) A solid hemisphere has a radius of 5.8 cm. Find its surface area. Take pi = 3.142.

Solution: Surface area of hemisphere= $3\pi r^2$ = (3 × 3.142 × 5.8 × 5.8) = 317.1 cm^2

2) A ball has a radius of **11 cm**. Calculate its surface area. (Use π = $\frac{22}{7}$)

Solution: Surface Area of the ball = $4\pi r^2 = 4 \times (\frac{22}{7}) \times (11 \times 11) = 1521.14\, cm^2$

3) A bowl (hemisphere) has a radius of **7 cm**. Find its total surface area. (Use $\pi = \frac{22}{7}$)

Solution:

A hemisphere is half a sphere + a circular base.

Curved Surface Area= $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2 = 2 \times (\frac{22}{7}) \times (7 \times 7) = 308\, cm^2$

Circular Base Area= $\pi r^2 = (\frac{22}{7}) \times 7 \times 7\, cm^2 = 154\, cm^2$

Total Surface Area= $308\, cm^2 + 154\, cm^2 = 462\, cm^2$

Answer: The bowl's total surface area is $462\, cm^2$.

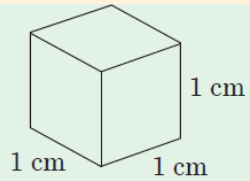
Application activity 13.7

1. Calculate the surface area of a sphere whose radius is:
   (a) 3.2 cm (b) 1.2 cm
2. Find the radius of a sphere whose surface area is:
   (a) 78.5 cm2 (b) 181 cm2
3. Find the total surface area of a solid hemisphere of diameter 10 cm.

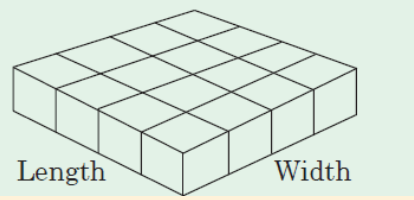
13.8 Finding volume of a prism

Activity 13.8

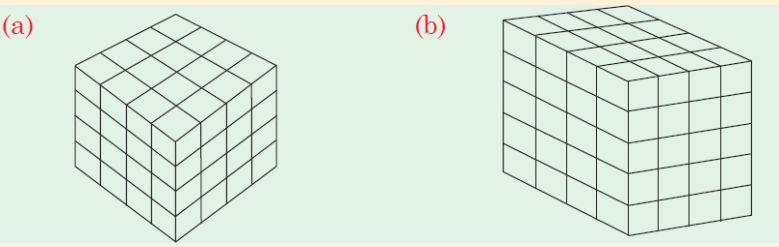
1. Review on the volume of cube and cuboid
   - Make several cubes like one shown below. This is a unit cube.



   - Make a layer like the one below using unit cubes



   i) How many cubes are there along the length?
   ii) How many cubes are there along the width?
   iii) How many cubes are there along the height?
   iv) Count the number of cubes in the layer.
   v) Calculate the number of cubes in the layer.



- By adding similar layers on top, make the following:

i) How many cubes are along the length?

ii) How many cubes are along the width?

iii) How many layers are in the stack? Explain.

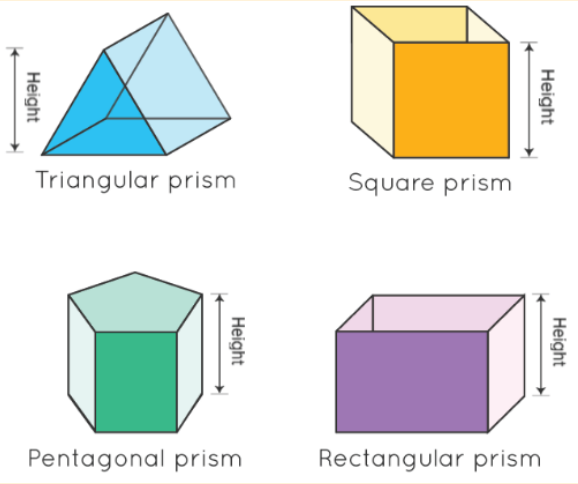
iv) How many cubes form each stack? This is the volume of the stack. Discuss your results.

v) Now compare your results in(iv) with the following product:

Cubes along length x cubes along width x cubes along height. This is the volume.

2. Explore the volume of a prism

– Observe each of the following prisms:



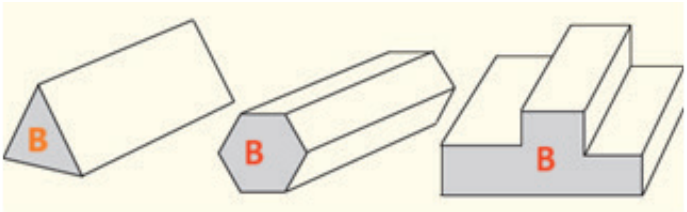
a) What is the shape of its base? How do you find the area of the base? Identify the height h.

b) Refer to the volume of a cuboid and discuss how you can find the volume of each prism.

Summary

Thus, the volume of a prism can be given as

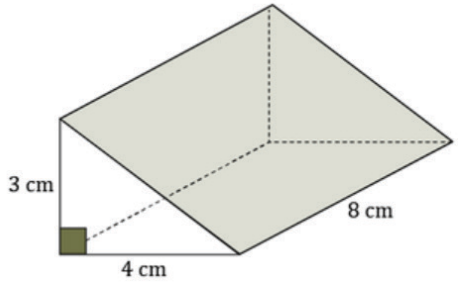
V = B × H where V is the volume, B base area, and H height of the prism.



Volume of a prism = Area of uniform cross-section × length (or height) of the prism.

Example:

A triangular prism has a right-angled triangular base with base 4cm and height 3cm.



If the length of the prism is 8 cm. Calculate the volume of the prism.

Solution

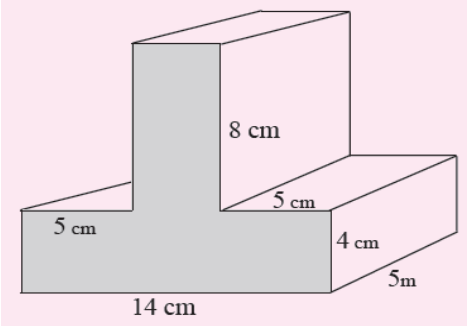
The base is a triangle: l= 4cm, h = 3cm

Length of the prism = 8cm

The area of the base B= $\frac{1}{2} \times 4cm \times 3cm = 6cm^2$

The volume is V = B × H= $6cm^2 \times 8cm = 48cm^3$

3. The following diagram is for a beam with the measurements in centimetres. If the length of the beam is 5 m and all angles are right angles, find its volume.



Solution

The end face forms our uniform cross-section.

The end-face may be taken as comprising of two rectangles.

The area of end-face is made of a rectangle 14cm by 4cm and the rectangle 8cm by 4cm.

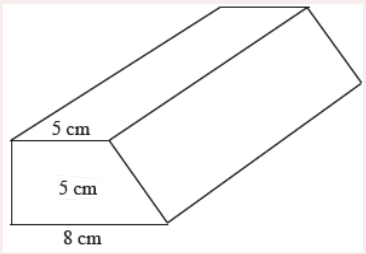
Therefore, Area of the base = (14 × 4 + 8 × 4) cm^2 = 88 cm^2

Length of beam = 5 m = 500 cm.

The volume of beam = Base area × length = 88 cm^2 × 500 cm = 44 000 cm^3

Application activity 13.8

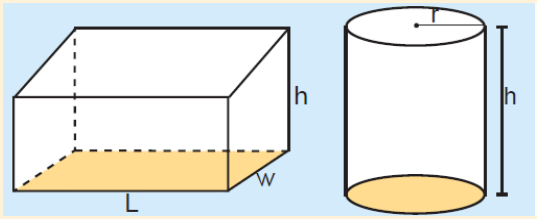
1. A wooden beam has a rectangular cross-section measuring 21 cm by 16 cm and it is 4 m long. Calculate the volume of the beam, giving your answer in cm^3.
2. A cylindrical container has a diameter of 200cm and a height of 20 cm. Using π=3.14, find how many litres of liquid it holds when full if $1dm^3 = 1l$ of water.
3. The volume of the prism with a trapezium base shown in the figure below is 1 170 . Find its length.

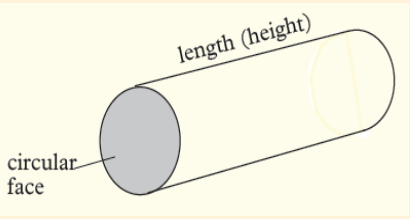


13.9 Volume of a cylinder

Activity 13.9

1. Observe the cuboid and cylinder below





a) Name the shape that makes the base of each solid.

b) What is the formula for calculating the area of the shape of the base?

c) Consider the formula for calculating the volume of a cuboid using the base surface area and discuss how you can find the volume of a cylinder. Then complete:

The volume of a cylinder = Base surface area x Height = ... x h

Summary

The volume of a solid with a uniform cross-section is given by:

Volume = Area of cross-section × length.

Thus, Volume of a cylinder = Area of the circular face × height = $\left(\pi \times r^2\right) \times h$

Example: A cylindrical fuel tank has a radius of 2 m. Its height is 6 m. Calculate its volume.

Solution: The base of a cylinder is a circle.

Radius (r) = 2 m; Height (h) = 6 m; **π (Pi) ≈ 3.14**

Volume = Base area x height = $\left(\pi \times r^2\right) \times h$ = 3.14 x 2x2 x6 =75.36 m^3

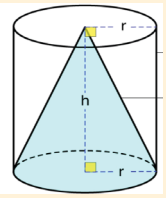
Application activity 13.9

1. Find the volume of each of the following cylinders.
   a) Radius 2.1 cm, height 0.3 cm.
   b) Radius 0.7 cm, height 4.5 cm.
2. A cylindrical container of diameter 15 cm and depth 20 cm is full of water. If the water is poured into an empty cylindrical jar of diameter 10 cm, find the depth of the water in the jar.
3. A cylindrical tank has a diameter of 5.0 m and contains 110 000 dm^3 of water. What is the height of the water in the tank?

13.10 Finding volume of a cone

Activity 13.10

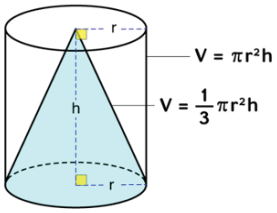
1. Construct a cone and a cylinder so that each solid has the same radius and same height.



2. Find the base area of the cone in (1) above.
3. Fill the cone with dry stuff like rice, flour, beans, maize etc.
4. Pour the content of the cone into the cylinder. How many times do you need to pour the content of the cone into the cylinder to fill it up?
5. What can you conclude about the volume of a cone basing on the volume of a cylinder with the same base and same height?

Summary

We need 3 cones to refill water in the cylinder with the same base and same height.



This means that the volume of a cone is the third of the volume of a cylinder with the same base and same height.

Volume of the cylinder with radius r and height h is

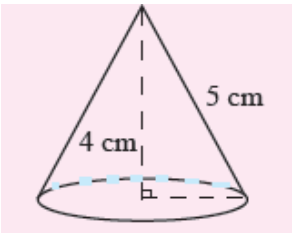
$$\text{Volume of a cylinder} = \pi r^2 \times h$$

Volume of *a* cone with the same radius r and height h is $\text{Volume of a cone} = \frac{1}{3} \times \pi r^2 \times h$

Examples:

1. Find the volume of *a* cone whose height and radius of the base are 4 cm and 3 cm, respectively. (Take π= 3.14).

Solution: r = 3 cm and h = 4 cm



Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.142 \times 3 \times 3 \times 4 cm^3 = 37.7 cm^3$

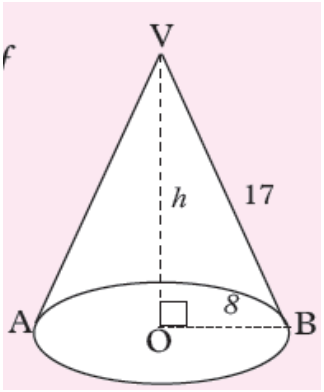
2. Acone has *a* radius of 8 cm and *a* slant height of 17 cm. Calculate the volume of the cone.

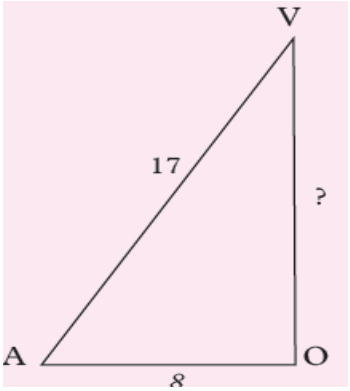
Solution: Let O be the centre of the base

OB = radius = 8 cm

VB = slant height = 17 cm. the height of the cone = 15 cm

Volume of the cone = ?





Assuming that this is *a* right cone, then ΔVOA = ΔVOB, right angled at O.

We find that VO = 15 cm, therefore, OV = height = 15cm

Area of the base = $\pi r^2 = 3.142 \times 8 \times 8$

Volume =($\frac{1}{3}$) x base area x height = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.142 \times 8 \times 8 \times 15 cm^3 = 1005.44 cm^3$.

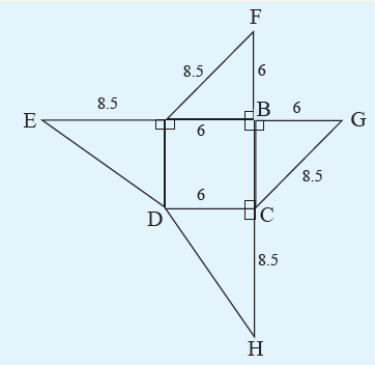
Application activity 13.10

1. Find the volume of the given cone. Take π= 3.14.
   a) Height 4 cm; area of base 15 cm^2
   b) Height 5 cm, radius 12 cm.
   c) Height 8 cm; base diameter 12 cm.
   d) Height 8 cm; base radius 3 cm.
   e) Slant height 9 cm; perimeter of base 12 cm.
2. Find the height of a cone whose base radius is 3.72 cm and whose volume is 143 cm^3.

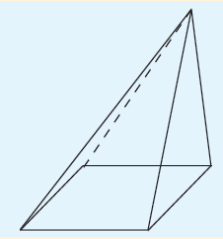
13. Volume of a pyramid

Activity 13.11

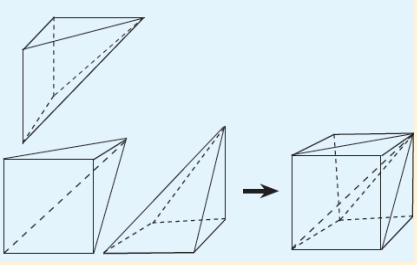
- Construct a net of a square based pyramid as shown in the following figure (All measurements are in centimetres):



- Cut the net out and fold the triangles up to form the pyramid as shown in the following shape:



- Rotate and arrange the three pyramids (each from each group member) to make a cube as shown in the following diagram:



What did you obtain?

Do you find a cube or a cuboid.What is the base of the cube obtained?

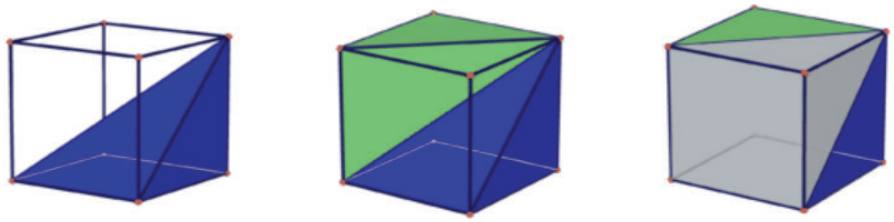
If you construct a cube whose base and height are equal to the base and height of the pyramids respectively.

When you open the base of the pyramid and the top of the cube, and fill the pyramid with dry substance such as rice, flour, soil and empty it into the cube, how many times will you need to empty the contents of the pyramid into the cube in order to fill it?

Refer to your findings to find the volume of the base of the pyramid.

Summary

It will take exactly 3 square pyramids to fill the cube with the same base and same side as the height of the pyramid. The image below shows each pyramid being added to the cube.



Therefore, 3 times the content of the pyramid = one times the content of the cube.

Volume of a pyramid = $\frac{1}{3}\times$ volume of cube.

Height of cube = height of pyramid.

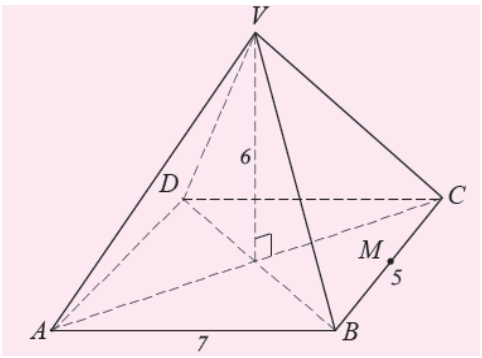
Since Volume of a cube = base area × height (Ah), then the volume (V) of a pyramid is given by

$\text{Volume of a pyramid} = \frac{1}{3}A \times h$ where A is the area of its base and h its height.

The base will depend on the type of pyramid you have.

Example

The figure below shows a pyramid on a rectangular base. Find its volume. The dimensions are in centimetres.



Solution

The base is a rectangle with l=7cm, w= 5cm.Height of the pyramid h = 6cm

Volume = $\frac{1}{3}A \times h = \frac{1}{3} \times l \times w \times h$ = ($\frac{1}{3}$) x 7cm x 5cm x 6cm= 70 cm^3.

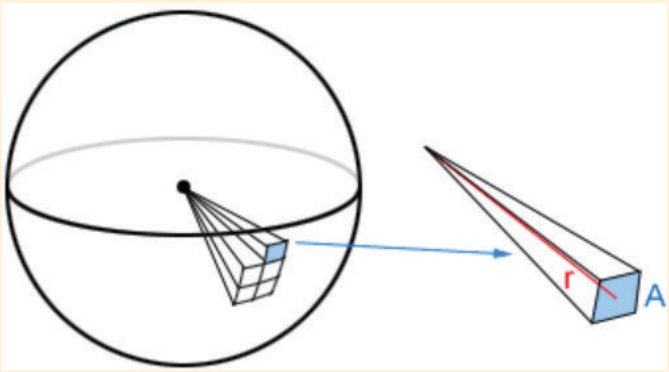
Application activity 13.11

1. Find the volume of each of the given right pyramid.
   a) Height 4 cm; square base of side 6 cm.
   b) Height 16 cm; right angled triangular base, base 6 cm and height 8 cm.
2. A pyramid whose height is 8 cm has a volume of 48 cm^3. What is the area of its base?
3. A pyramid has a square base of side 5 cm. What is its height if its volume is 100 cm^3?

13.12 Finding volume of a sphere

Activity 13.12

Obtain an orange and a knife. Cut out a very small squared area (A) from the surface to the centre of the orange as shown below:



a) If the solid formed by joining the vertices of A to the centre O is a small 'pyramid. How can you find its volume?

b) Consider that the sum of areas for all squares taken on the sphere is the total surface area of the sphere, how can you find the Volume of the sphere? Use the total volume of the pyramids found in (a).

Summary

The volume of the sphere can be approximated using the volume of small pyramids with height **h** equal to the radius **r** of the sphere and base taken from the surface of the sphere

The volume of each pyramid is $\frac{1}{3}A \times h = \frac{1}{3}A \times r$ where A is the area of the small surface taken on the sphere.

When we consider the entire sphere, the total area is $4\pi r^2$.

Therefore, The volume of the sphere $\frac{1}{3}\times 4\pi r^2 \times r = \frac{1}{3}\times 4\pi \times r^3$

Volume of a sphere $V = \frac{1}{3}\times 4\pi \times r^3$

Examples

1. Calculate the volume of a sphere with radius of 3cm. (use π= 3.14)

Solution: Radius r = 3cm. Pi = 3.14

$$V = \frac{1}{3}\times 4\pi \times r^3 = \frac{4\times 3.14\times 3\times 3\times 3}{3} cm^3 = 113.04\, cm^3$$

The volume of the sphere is 113.04 cm^3

2. Find the volume of a solid hemisphere whose diameter is 10 cm.

Solution: Diameter = 10cm, the radius = 10cm ÷2 = 5cm

$$V = \frac{1}{3}\times 4\pi \times r^3 = \frac{4\times 3.14\times 5\times 5\times 5}{3} cm^3 = 261.67\, cm^3$$

The volume of the hemisphere is 261.67 cm^3 /2 = 130.83 cm^2

Note: You can use volume of hemisphere = $\frac{2}{3}$ × π × r^3. The answer remains the same.

Application activity 13.12

1. Calculate the volume of a sphere with the following radius (use π= 3.14)
   (a) 12 cm (b) 4 cm
2. Find the volume of a sphere whose surface area is 21.2 cm^2

13.5. End of unit assessment

13.5.1 Questions

Assessing knowledge and understanding:

Circle the correct answer for each question.

1. What is the area of a circle with radius 7 cm? (Use π= $\frac{22}{7}$)
   a) 44 cm^2 b) 154 cm^2 c) 308 cm^2 d) 616 cm^2
2. The volume of a cylinder is calculated using:
   a) V= π × r × h b) V= $\frac{1}{3}$. π. R. h c) V= 4.π.r.r.r d) V = l × w × h

Assessing skills:

3. Surface Area Calculations

a) Find the total surface area of a closed cylinder with radius 3 cm and height 10 cm. (Use π=3.14)

b) A pyramid has a square base of side 6 cm and a slant height of 5 cm. Calculate its surface area.

c) Calculate the surface area and the volume of each of the following solids:

i) Cylinder, radius 4.8 m, height 6.9 m

ii) Rectangular prism, 4.3 cm by 6.8 cm by 12.8 cm

iii) Sphere, diameter 8.25 cm

d) The base of a rectangular tank measures 62 cm by 130 cm. If it is 40 cm high, find the area of metal sheet required to make it.

4. Volume Problems

a) A rectangular prism measures 8 cm × 5 cm × 6 cm. Find its volume.

b) Calculate the volume of a sphere with radius 4.5 cm. (Use π=3.14)

c) A classroom measures 10 m × 9 m × 4 m. Each person in the room requires 8 m^3 of air to be comfortable. How many students should learn in the room so that everybody is comfortable?

5. A conical tent made in canvas has a radius of 7 m and a height of 10 m.

a) How much canvas is needed to make the tent? (Ignore the base.)

b) What is the volume of air inside the tent?

Assessing attitudes and values

6. Explain how understanding surface area and volume helps in:

a) Wrapping a gift (e.g., calculating wrapping paper needed).

b) Designing a water tank (e.g., determining capacity).

7. A student says, "A cone and a cylinder with the same height and radius have equal volumes." **Is this correct? Justify your answer with calculations.**

8. A storage container consists of a cylinder (radius 5 cm, height 12 cm) topped by a hemisphere (same radius).

a) Find the total surface area.

b) Calculate the total volume

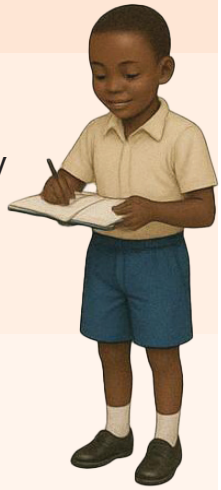
13.5.2 Presentation of projects on solids (3D objects) made using nets.

UNIT 14

DATA HANDLING

Key unit competence: You will be able to collect, represent and interpret data in order to solve a problem.

Learning objectives: By the end of this unit, you should be able to:



Knowledge and understanding:

- Explain when it is appropriate to use a tally and how to obtain frequency distribution.
- Explain how to use pie charts to represent proportions of data.
- Interpret line graphs as representation of data.

Skills:

- Decide what data to collect to answer a question.
- Collect data using a table and tally.
- Represent data in a bar chart or pie chart where the total frequency is a factor of 360^0.
- Interpret representations of data to answer a question.

Attitudes and values:

Appreciate the role of organised data as the support for taking decision.

14.0 Introduction

Data is information or facts that we collect, like numbers, words, pictures, or sounds. It helps us to learn and to make decisions.

Examples of data include **Numbers, Words such as** names and favorite color, and p**ictures.**

Data are helpful in knowing how things are taking place. They are also helpful in good planning. People can know things that happened long ago due to stored data or information. Businesspeople, schools, government record data or information to be used when making decisions.

Introductory Activity

a) Do you think collecting data and keeping them is important in daily life?

b) Which kind of data can a person collect? Give an example of data which can be collected in the environment.

c) Is collecting data, organizing data, and interpreting them useful to make a family budget?

d) Do you think data handling is important to you? Give 2 reasons.

14.1 Collecting the data to investigate a question

Activity 14.1

List the marks obtained by every learner in your class in the end of unit assessment.

- Group the same marks and record their number.
- Use small lines to represent the number of pupils with the same marks.
- Complete a table showing a mark and number of pupils obtained each mark.
- Explain the data in the table to the class.

Summary

Data collection is the organized process of gathering and measuring information on targeted variables to answer research questions.

Example

Below is mass in kg of P6 learners who underwent a Body Mass Test. 28, 30, 28, 33, 35, 40, 40, 28, 30, 30, 42, 40, 35, 40, 40, 40, 33, 30, 28, 30, 30, 30, 33, 40, 33, 33, 35, 40, 35, 40, 35, 35.

a) Use tally marks to organise the data above in the table

b) How many learners underwent the Body Mass Test?

c) Which mass was gotten by most pupils?

Solution a) Follwow the table below:

Mass in kg	Tally	Frequency (Number of learners)
28	𝍸	4
30	𝍸 ||	7
33	𝍸	5
35	𝍸 |	6
40	𝍸 𝍸	9
42	|	1
Total		32
b) 32 learners underwent the Body Mass Test		
c) The mass of 40kg was gotten by most pupils.		

Application activity 14.1

1. The following marks were obtained by a P 6 class in a Mathematics Examination.

   55, 68, 66, 66, 83, 57, 70, 73, 70, 73, 55, 66, 68, 83, 66, 73, 72, 57, 73, 83, 57, 71, 65, 78, 74, 56, 68, 71, 83, 85, 83, 68, 70, 71, 55, 70, 61, 63, 67, 72, 70, 74, 59, 60.

   a) Use tallies to organise the data in a table.

   b) How many learners did the examination?

2. Kayitesi bought kg of maize flour for several days as shown below: 5, 3, 6, 5, 6, 3, 1, 2, 1, 2, 3, 4, 5, 6, 3, 4, 4, 3, 5, 2, 1, 1, 2, 3, 4, 5, 6, 4, 5, 6, 6, 6, 5, 4, 4, 2, 3, 4, 3, 4, 2, 2, 4, 1, 4, 2, 3, 4, 4, 3, 2, 4, 6, 5, 4, 6, 4, 5, 4, 5.

   a) Organise the data in a table.

   b) State the number of days she bought maize flour.

   c) On how many days Kayitesi bought 1 kg?

14.2 Explore tally and frequency tables

Activity 14.2

Study the table below:

Marks obtained in a Science test out of 50	**Tally**	**Frequency:** Number of pupils
10	|	
20	𝍸 |	
30	𝍸 |||	
40	||||	
Total		19

a) Write the number of pupils who obtained each mark.

b) If all learners did the test, how many learners are in the class?

c) Which mark was obtained by small number of pupils?

d) What is an other name of "the number of pupils" in this question.

Summary

Frequency is the number of times a quantity appears.

Tallies are also used to show the occurrences or frequencies.

Tallies are written with strokes to show how many times a number appears.

Example

The table below shows the marks obtained by P.6 learners in a Mathematics exam by percentage in a school. Study it and answer the questions that follow

Marks	Tally	Frequency (Number of learners)
96	𝍸	4
95	𝍸	5
93	𝍸 ||	7
84	𝍸 𝍸 ||	12
81	𝍸 𝍸 𝍸 |||	18
72	𝍸	5
70	|||	3
68	𝍸 |	6
		60

a) How many learners did the exam?

b) Which mark was scored more by learners?

c) What was the least mark scored?

d) How many learners scored 70%?

e) Find the number of learners who got above 80%.

Solution

a) 60 learners did exam

b) 81% mark was scored most by learners because it has 18 frequencies

c) 70% was the least mark scored.

d) 3 learners scored 70%.

e) There are 46 learners who got above 80%.

Application activity 14.2

1. Mr. Kanyamashyamba planted different types of saplings. The data is shown in the table below. Study it and answer the questions that follow.

Saplings	Tally	Frequency
Eucalyptus	𝍸 𝍸 𝍸 𝍸	20
Pine	𝍸 𝍸 𝍸 |	16
Spruce	𝍸 𝍸 𝍸 |	11
Cypress	𝍸 𝍸 𝍸 𝍸	20
Podo	𝍸 𝍸 𝍸 𝍸 |||	23
Cedar	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
Total		

a) How many cypress and podo saplings were planted?

b) Which type of saplings had the same number?

c) Find how many more cedar saplings than spruce saplings were planted?

d) How many saplings were planted altogether?

2. The table below shows the age of learners in P. 6 class. Use it to answer the questions that follow:

Age	Tally	Frequency
10 years	𝍸	
11 years	𝍸 𝍸 𝍸 𝍸 |	
12 years	𝍸 𝍸 𝍸 |||	
13 years	𝍸 ||	
Total		

a) How many learners are 12 years old?
b) How old are most of the learners?
c) How many learners are in the class?

14.3 Representing the data in a bar chart

Activity 14.3

The table below shows items that a country exports as well as the contribution of each item to the total item value.

Export item	Tea	Sorghum	Flowers	Beans	Maize
Percentage of item	20	45	15	12	8

a) What is a bar graph?
b) Represent the above information in a bar graph.
c) Make a presentation to the class.

Summary

A bar graph is a graph consisting of vertical or horizontal bars whose lengths are proportional to the amounts or quantities of data represented.

A bar graph is also called bar chart or bar diagram.

Always choose a suitable scale for the vertical axis.

Example

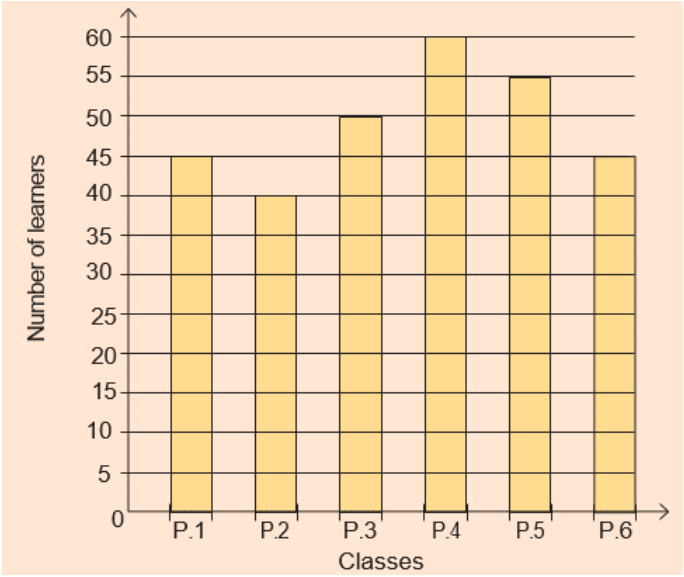
The table below shows number of learners per class in a school in Rwanda

Class	P1	P2	P3	P4	P5	P6
Number of learners	45	40	50	60	55	45

a) Represent the above information in a bar graph.
b) How many pupils were in the school?

Solution

a) A bar graph showing number of learners per class in a school



b) 45 + 40 + 50 + 60 + 55 + 45 = 295. The school has 295 learners.

Application activity 14.3

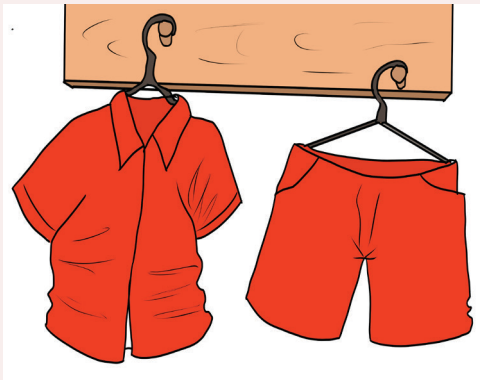
1. The table below shows the number of litres of milk collected on a farm in a week.

Day	Mon	Tue	Wed	Thurs	Fri	Sat	Sun
Milk in litres	50	60	45	75	40	25	65

a) Represent the above information in a bar graph.

b) How many litres of milk were produced in the week?

2. P. 6 learners were asked to pick clothes of their best colour.



The results were recorded as follows: G = green, R = red, B = blue, O = orange and Y = yellow:

G Y R B G G R Y R O G R Y Y G G Y R B G G R Y R O G R Y Y G G Y R B G G R Y R O G R Y Y G Y R G Y G R R O O Y G O R R R.

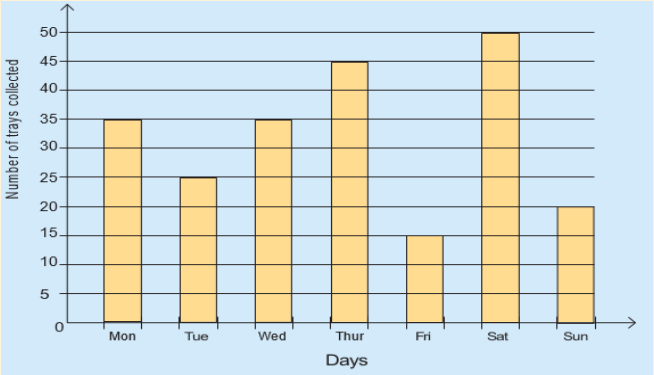
(a) Organise the data in a table. (b) Represent the data in a bar chart.

14.4 Interpreting the data in a bar chart

Activity 14.4

Study the bar graph below. It presents the number of boys collected by Mr. Kagabo per day. Answer the questions that follow:

Number of trays collected per day



- How many days are shown on the graph?
- What is observed on the vertical axis?
- Find the number of trays collected in the whole week.
- On which day did Mr. Kagabo collect fewer trays of eggs?

Summary

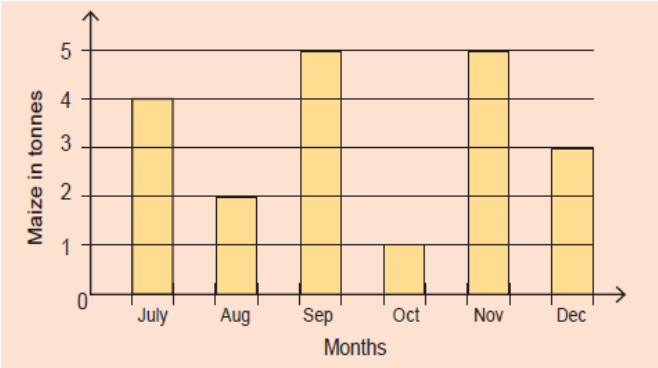
The title of a bar chart tells the subject on which data were collected.

Always take note of what is represented on the vertical and horizontal axes.

The vertical axis is divided into equal parts.

The reading on each part of the vertical axis is the vertical scale.

Example : Tonnes exported by Mr. Muhire



a) In which months did Mr. Muhire export the same number of tonnes?

b) How many more kg of maize were exported in November than in August?

c) Find the total tonnes of maize exported.

Solution

a) In the months of September and November, he exported 5 tonnes.

b) November = 5 tonnes and August = 2 tonnes,

(5-2) tonnes=3tone but 1 tonne=1000kg then 3tonnes==3x1000kg=3000kg

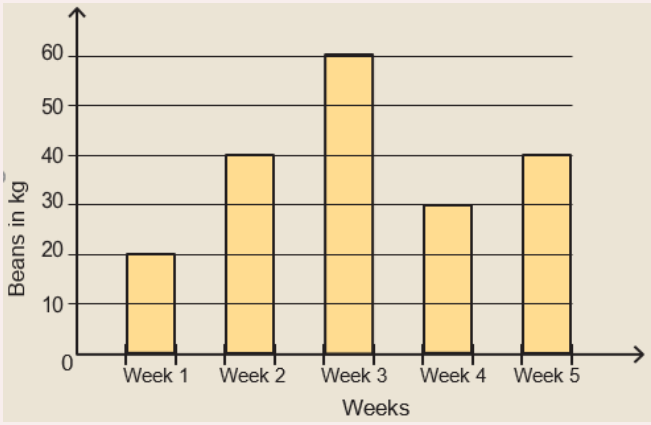
Therefore 3,000 kg more were exported in the month of November than in August.

c) Total tonnes exported = (4 + 2 + 5 + 1 + 5 + 3) tonnes = 20 tonnes

Application activity 14.4

Study the graph and answer the questions that follow:

1. Beans in kg sold at a shop in 5 weeks



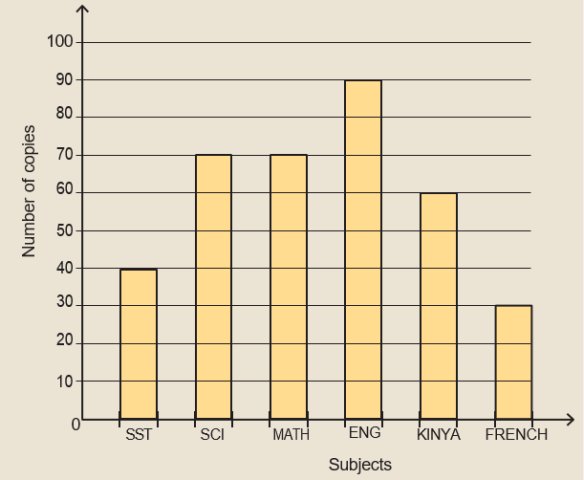
a) In which weeks did the shopkeeper sell the same quantities of beans?

b) In which week did the shopkeeper sell the highest quantity of beans?

c) Find the total kilogram of beans sold in the 5 weeks

d) If each kg of beans costs 700 Frw, how much did the shopkeeper get from the sale of beans in the five weeks?

2. The bar graph below shows the number of textbooks bought by a primary school in 2024. Study it and answer the questions that follow if SCI means Sciences:



a) Which subject has the highest number of copies?

b) Which subjects have the same quantity?

c) How many more English books were bought than Social Studies books?

d) If each Mathematics book costs 5,500 Frw, how much money was spent on the Mathematics textbooks?

e) How many textbooks were bought altogether?

14.5 Representing Data in Pie Charts

Activity 14.4

P6 learners at a school performed as follows in the end of year examination; Division I: 18 learners, Division II: 25 learners, Division III: 12 learners and Division IV: 5 learners.

a) Find the total of learners

b) Get the fraction of each division.

c) Multiply each fraction by 360 degrees to get the degrees of each division.

d) Draw a circle and divide it into sectors by using the corresponding degrees.

e) What special name is given to the above circle?

Summary

A pie chart is a circular chart divided into slices or sectors. Each slice/sector represents a part or a category of a whole. Each part can be represented in degrees.

The quantity represented in each sector corresponds to its fraction out of 360degrees. It is also called a circle graph.

First find the total of items if it is not given. Write each item as a fraction of the total then multiply by 360 degrees. Fractions add up to 1, degrees add up to 360 degrees while percentages add up to 100%.

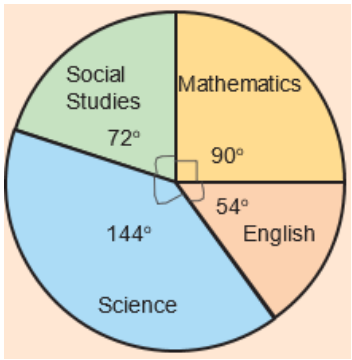
Example

In a P 6 class, 15 learners like Mathematics, 12 learners like Social Studies, 24 learners like sciences and 9 learners like English. Represent the information in a pie chart.

Solution

Total learners = (15 + 12 + 24 + 9) = 60 learners. Find the sector of every subject. Use the table below. Use a protractor to draw the angle sectors in the pie chart. Use the skill of drawing angles.

Subject	Number of learners	Fractions	Percentages	Degrees
Mathematics	15	$\frac{15}{60}=\frac{1}{4}$	$\frac{1}{4}\times 100 = 25\%$	$\frac{1}{4}\times 360 = 90^{\circ}$
Social Studies	12	$\frac{12}{60}=\frac{1}{5}$	$\frac{1}{5}\times 100 = 20\%$	$\frac{1}{5}\times 360 = 72^{\circ}$
Science	24	$\frac{24}{60}=\frac{2}{5}$	$\frac{2}{5}\times 100 = 40\%$	$\frac{2}{5}\times 360 = 144^{\circ}$
English	9	$\frac{9}{60}=\frac{3}{20}$	$\frac{3}{20}\times 100 = 15\%$	$\frac{3}{20}\times 360 = 54^{\circ}$
Total	60	1	100%	360°



Note: It is possible to indicate percentages in the pie chart instead of angles.

Application activity 14.5

1. The table below shows percentage expenditure of a publishing company. Study the data and represent it in a pie chart.

Item	Printing cost	Transportation cost	Paper cost	Binding	Royalty	Promotion cost
Percentage expenditure	20%	10%	25%	20%	15%	10%

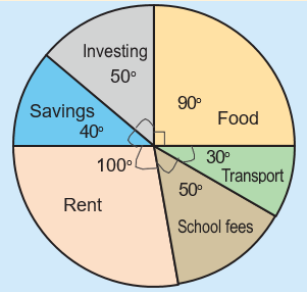
2. The table below shows how 120 learners participated in different games. Represent the information in a pie chart

Game	Football	Netball	Volleyball	Tennis	Rugby
Number of pupils	40	15	20	10	35

14.6 Interpreting the data in pie charts to draw a conclusion

Activity 14.6

The pie-chart represents the monthly expenditure of Mrs. Muziranenge's salary. Study it and answer the questions that follow given that the fraction of transport is $\frac{30^0}{360^0}$.



If 240,000 Frw is spent on transport, how much does she earn?

How much more money is spent on rent than on savings?

Present your working to the class.

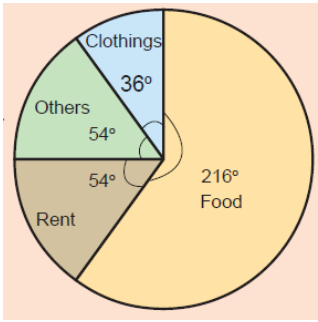
Summary

To get the number of items for each sector, multiply the fraction for that sector by the total number.

To get percentage for each sector, multiply the fraction for the sector by 100%.

Example

Byiringiro spends his salary of 43,000 Frw as shown in the pie chart. Study it and answer the questions that follow.



a) How much is spent on food?
b) Which items does he spend on the same amount?
c) What is the percentage of money spent on food?
d) How much more money does he spend on food than on others?

Solution

a) The money spent on food =

$$\frac{216^0}{360^0}\times 43000 = 6\times 4300 = 25{,}800\text{Frw}$$

b) He spent the same money on rent and others because they have 54^0 =

$$\frac{54^0}{360^0}\times 43000Frw = \frac{3}{2}\times 4300Frw = 3\times 2150Frw$$

= 6450Frw

c) The percentage of money spent on food=

$$\frac{216^0}{360^0}\times 100\% = 6\times 10\% = 60\%$$

d) The money he spent on food than on others =

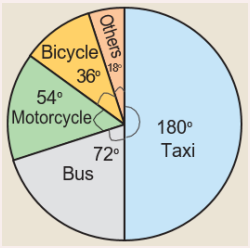
$$\frac{54^0}{360^0}\times 43000Frw = \frac{3}{2}\times 4300Frw = 3\times 2150Frw$$

=6450Frw

The difference = (25,800-6,450) FRW= 19,350 Frw is more spent on food than on other items.

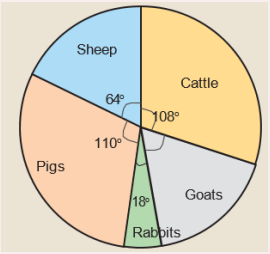
Application activity 14.6

1. The pie chart shows the means of transport that people use in an area.



   a) Which means of transport is used by most people in the area?
   b) If 6,000 people use motorcycles, how many people are in the area?
   c) How many more people use taxis than bicycles?

2. The pie chart below shows animals Mr. Mugwaneza has on his farm



   a) Find the percentage of goats reared on the farm.
   b) If there are 300 animals on the farm, how many pigs are there?
   c) If each rabbit is sold at 15,000 Frw, how much can he get from selling all the rabbits on his farm?

14.7 Collecting and representing data in a bar chart or a pie chart

Activity 14.7

A class of 30 students was surveyed about their favorite fruit. The results are as follows:

- Apple: 10 students
- Banana: 8 students
- Orange: 6 students
- Mango: 4 students
- Grapes: 2 students

Find out

a) Summarize the data in a frequency table.
b) Represent the data using a bar chart
c) Represent the data using a pie chart.

Summary

To effectively summarize data using tables and visualizations like bar charts or pie charts, follow these steps: First, collect the data, then categorize it and count occurrences for each category, creating a frequency table. Finally, represent this data visually using a bar chart for comparing different categories or a pie chart to show parts of a whole.

Example

A teacher asks 30 students about their favorite school subjects. The responses are collected in no particular order: Math, Science, English, Math, History, Science, Math, English, Art, Math, Science, History, English, Math, Art, Science, Math, English, History, Math, Science, English, Art, Math, Science, History, Math, English, Science, Math, Art

Tasks:

a) Summarize the data in a frequency table.

b) Represent the data using a bar chart.

c) Represent the data using a pie chart.

Solution

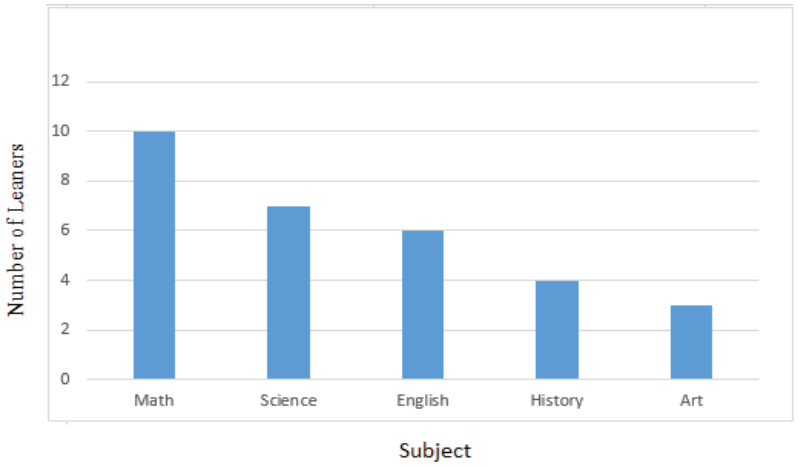
The frequency table

Subject	Tally	Frequency (Number of learners)
Math	𝍸 𝍸	10
Science	𝍸 ||	7
English	𝍸 |	6
History	|||	4
Art	||	3
Total		30

a) The representation by a bar chart

Subject	Math	Science	English	History	Art
Number of learners	10	7	6	4	3

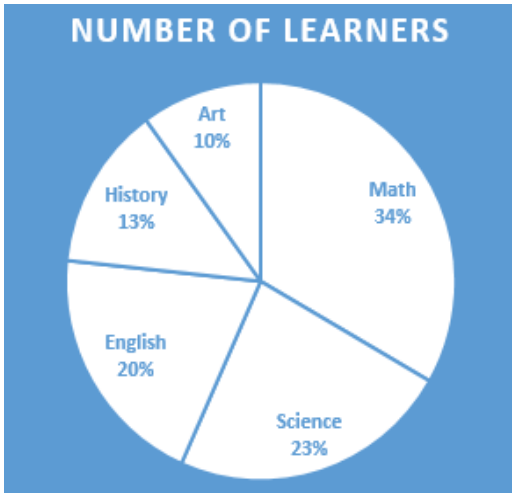
With the table above, represent the bar chart.



b) Representation the data using a pie chart

Subject	Number of learners	Fractions	Percentages	Degrees
Math	10	$\frac{10}{30}=\frac{1}{3}$	$\frac{1}{3}\times 100$ = 34%	$\frac{1}{3}\times 360$ = 120°
Science	7	$\frac{7}{30}$	$\frac{7}{30}\times 100$ = 23%	$\frac{7}{30}\times 360$ = 84°
English	6	$\frac{6}{30}=\frac{1}{5}$	$\frac{1}{5}\times 100$ = 20%	$\frac{1}{5}\times 360$ = 72°
History	4	$\frac{4}{30}=\frac{2}{15}$	$\frac{2}{15}\times 100$ =13%	$\frac{2}{15}\times 360$ = 48°
Art	3	$\frac{3}{30}=\frac{1}{10}$	$\frac{1}{10}\times 100$ =10%	$\frac{1}{10}\times 360$ = 36°
Total	30	1	100%	360°

The pie chart



Application activity 14.7

1. In a class of 40 students, the following preferences for subjects were recorded:

- Mathematics: 12 students
- Science: 10 students
- English: 8 students
- History: 6 students
- Geography: 4 students

Tasks:

a) Summarize the data in a frequency table.

b) Represent the data using a bar chart.

c) Represent the data using a pie chart.

2. A group of 25 students reported the number of hours they study daily:
   - 1 hour: 5 students
   - 2 hours: 10 students
   - 3 hours: 7 students
   - 4 hours: 3 students

Tasks:

a) Summarize the data in a frequency table.

b) Represent the data using a bar chart.

c) Represent the data using a pie chart.

14.6. End unit assessment

14.6.1 Questions

Assessing knowledge and understanding:

1. What is a frequency table?
2. How do you calculate the angle for a sector in a pie chart?
3. **Identify:**
   a) In a bar chart, what does the vertical axis represent?
   b) If a pie chart has a sector of 90°, what fraction of the whole does it represent?

Assessing skills:

4. **Below are heights in cm of P6 learners in their study of measurement.**

125	128	118	130	142	125	125	118	118	128	134	128	118	125
130	128	130	142	125	128	130	128	130	132	128	132	130	125
142	128	130	128	130	128	140	134	130	128	132	138		

   a) Use tally marks to organise the data above in a table.

   b) How many learners were measured?

   c) Which height is the most common (with more learners)?

5. P6 learners were asked to state the food type they like most. 9 learners like cassava, 10 learners like rice, 5 learners like Irish potato, 6 learners like banana.

Represent the information in a pie chart.

Assessing attitudes and values

6. Why might incorrect data in a hospital's patient records be dangerous? Give two reasons.
7. A shopkeeper says, "Bar charts are useless for my business." Do you agree? Justify your answer.
8. Which data representation (tables, bar charts, pie charts) do you find most useful? Why?

14.6.2 Presentation of projects conducted on data collection, organisation and presentation.

UNIT 15

LIKELIHOOD OF EVENTS

Key Unit Competence: You will be able to compare and order events based on their of likelihood.

Learning objectives: By the end of this unit, you should be able to:

Knowledge and understanding:

- Explain that random events have different chances of happening.
- Illustrate key probability terms (e.g., certain, likely, unlikely, impossible) with examples.

Skills:

- **Use** the language of chance (e.g., "certain," "likely," "unlikely", "impossible") to describe real-world events.
- **Compare** and **order** events based on their likelihood (from least to most probable).

Attitudes and values

- Recognize that random events cannot be predicted with absolute certainty.
- Appreciate the status of chance in making informed decisions.

15.0 Introduction

In this unit, we will learn about types of events in probability: **impossible**, **equally likely**, and **certain**. Impossible events never happen, this is for example "a human flies without wings". Equally likely events have the same chance of occurring, this is for example "getting heads or tails "when you flip a fair coin. Certain events always happen, for example "the sun rises every morning".

These ideas aren't just for mathematics. They help us in real life too! For example, before any Football match starts, the spectators are not sure who will win. The chance of winning the game is equal at the beginning. The two teams are likely to win. At the end of the game, different possible outcomes or results may occur. This means that understanding chance makes us better at predicting what could happen.

Introductory Activity

a) Look outside the classroom. Can you be sure it will rain today? Is raining impossible or it is equally likely to rain?

b) In a football match, do you think both teams will finish with the same score? Is it likely to happen, or is it impossible? Explain your answer.

15.1 Vocabulary of chance: impossible, certain.

Activity 15.1

Toss a fair coin or a fair die many times. Try again and see the results.

Here are flash cards on which you can read: **certain, possible and impossible.**

1. Use the appropriate flash card to name the following event related to tossing a coin
   i) Getting a head. _____
   ii) Getting a head and tail at the same time. _____
   iii) Getting a head or a tail. _____
2. Use the appropriate flash card to name the following event related to tossing a fair die
   i) Getting a number less than 7 _____
   ii) Getting an even number _____
   iii) Getting an odd number. _____
   iv) Getting a number greater than 6 _____
   v) Getting number 4 or 3 _____

Summary

Impossible event: An event that cannot happen.

Example: Getting a tail and a head at the same time when you toss a fair coin once.

Certain event: An event that will definitely happen.

Example: The sun rises each morning; it is certain to happen.

Considering that when the event is impossible, the chance of happening is 0. When the event is certain, the chance of it to occur is 100% or 1. On the line of chance we have:

Application activity15.1

1. From the following events, show which are impossible and which are certain.
   a) The sun will rise tomorrow.
   b) A fish flying to the moon by itself.
   c) Rolling a 7 on a standard 6-sided die.
   d) Dropping a stone will make it fall to the ground.
2. Complete by: **certain, possible or impossible.**
   a) People can live without water it is _____
   b) In a game of a well shuffled playing cards, selecting a black card is _____as selecting a red card.

15.2 Vocabulary of chance: equally likely/even chance, unlikely, likely

Activity 15.2

1. Roll one die. Remember that the possible outcomes are 1, 2, 3, 4, 5 and 6.
   - How many ways are there of a 2 showing up? Is the event "showing a 2"unlikely or it has even chance?
   - How many ways are there of a 7 showing up? Is the event showing a 7 unlikely or impossible?
2. Now, toss a fair coin. Observe that a coin has 2 faces, the head and the tail. Show these faces to each other.

Discuss the likelihood of getting a tail when a coin is tossed.

Work in pairs and play the game of tossing a coin. Suppose that the winning face is the tail. Can you predict who will win after one toss?

Now, is having a tail **equally likely** or unlikely to happen?

Now formulate other events which are **certain, likely, equally likely, unlikely, impossible.**

3. Suppose that you roll a fair die. Complete with **certain, likely, equally likely, unlikely or impossible.**

Example: Getting an odd number: it is **equally likely** because the number of odd numbers (1,3,5) is equal to the number of even numbers (2,4,6) they are all three.

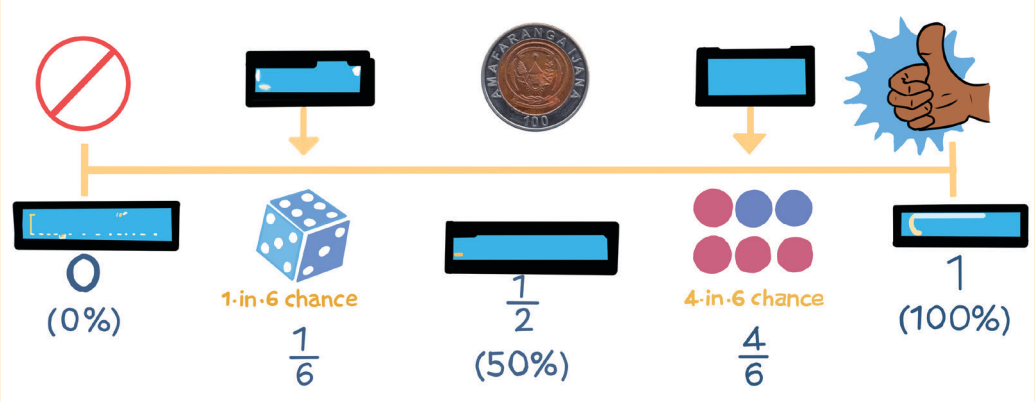
i) Getting a number greater than 6: _____

ii) Getting the number 2: _____

iii) Getting an even number: _____

iv) Getting a number less than 6: _____

4. Now, refer to your results in (3) and place the cards of chance with **certain, likely, equally likely, unlikely and impossible** for each event on the following probability line. Note that one card needs to replace the blue card:



Summary

The following is the explanation of each term related to probability, along with an example event for each:

- **Certain:** An event that is guaranteed to happen (chance is 1 or 100%).

  Example: "The sun will rise tomorrow."

- **Likely:** An event that has a high chance of happening but is not guaranteed (chance is greater than 50% but less than 100%).

  Example: "It will likely rain this afternoon because the sky is cloudy."

- **Equally likely (Even chance):** An event that has the same chance of happening as not happening (chance equal to 50%).

  Example: "Flipping a fair coin and getting heads."

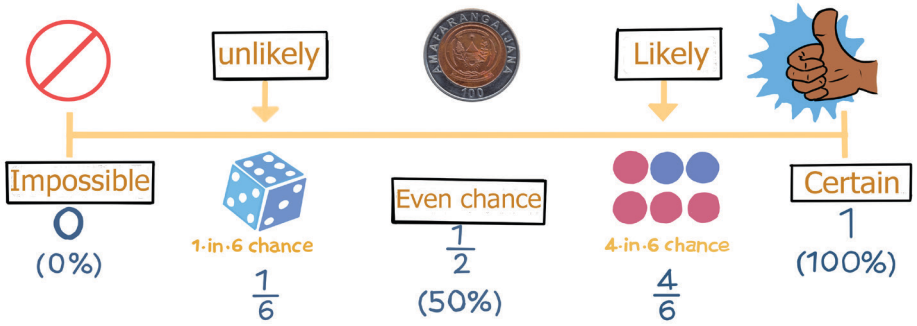
- **Unlikely:** An event that has a low chance of happening but is still possible (chance less than 50% but greater than 0%).

  Example: "Winning the lottery with a single ticket."

- **Impossible:** An event that cannot happen under any circumstances (chance= 0 or 0%).

  Example: "A human being will grow wings and fly without any equipment."

On ***a probability line*** we have:



Examples

Discuss the likelihood of the following events:

1. Getting a head when you toss a fair coin. **Equally likely**
2. It will rain in Kigali this year. **Certain**
3. A woman will give birth to a boy. **Likely**
4. The sun will rise tomorrow. **Likely**
5. It will rain tonight - **equally likely**.
6. A lion eats grass - **impossible**.

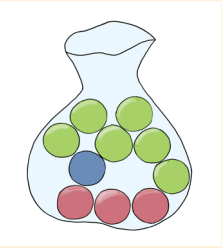
Application activity 15.2

1. Discuss the likelihood of the following events:
   a) You will get anumber less than seven when you throw a die.
   b) You will get a total of 1 when you throw two dice.
   c) Our teacher will become the president.
   d) I was born yesterday
2. Explain the following event used in probability and provide example of related event:
   a) Impossible
   b) Certain
   c) Equally likely
   d) Likely
   e) Unlikely
3. Create your own statements and associate them with vocabulary of chance.

15.3 Using expected outcomes of experiment to explore less likely and more likely

Activity 15.3

You have a bag with counters with different colours: 6 green, 3 red and 1 blue.



a) Pick and pull out a counter 20 times (with replacement).
   Record the colour of the counter each time.
b) Which colour is more likely to be picked?
c) Which colour is less likely to be picked?

Summary

We use **"more likely"** and **"less likely"** to compare the chances of different outcomes. Here's how they work:

More Likely: Used when an event has a **higher chance** of happening compared to another event or a baseline.

Example:

"If you study regularly, you are more likely to pass the exam.» (Higher chance than if you don't study.)

“Smokers are more likely to develop lung cancer than non-smokers.

“ **Less Likely:**Used when an event has a **lower chance** of happening compared to another event or a baseline.

Example:

“If you don’t practice, you are less likely to improve.» (Lower chance than if you practice.)

“People who exercise daily are less likely to have heart disease.

Example:

Experiment: Pick a ball from a bag with 4 red balls and 1 blue ball.

Picking **red** is **more likely** (4 out of 5) and Picking **blue** is **less likely** (1 out of 5).

Application activity 15.3

1. I pull a marble from a bag with 9 green and 1 yellow marble.
   - What is the expected outcomes?
   - Is picking a green certain, less likely, more likely, unlikely or impossible?
2. Let us consider an experiment of flipping a regular dice once. Complete by less likely, more likely or equally likely the following statements:
   a) “Getting a number greater than one” .....
   b) “Getting an even number”.....
   c) “Getting a divisor of six”:.....
   d) “Getting a prime number”:.....

15.4 Determining the likelihood of events

Activity 15.4

1. Toss a fair coin. Observe that a fair coin has 2 faces, the head and the tail. Show these faces to each other.
   a) Discuss the likelihood of getting a tail when a coin is tossed.
   b) Work in pairs and play the game of tossing a coin. One will choose Tail the other will choose the head. Suppose that the winning face is the tail.
      i) Can you predict who will win after one toss?
      ii) Is having a tail **equally likely** or unlikely to happen?
2. Roll one regular six-sided die. Remember that the possible outcomes are 1, 2, 3, 4, 5 and 6.
   a) How many ways are there of a 2 showing up? Is the event “showing a 2” unlikely or it has even chance?
   b) How many ways are there of a 9 showing up? Is the event showing a 9 unlikely or impossible?

Summary

- **Certain Event**: Will always happen (e.g., the sun will rise tomorrow).
- **Likely Event**: High chance of happening (e.g., rain in the rainy season).
- **Equally Likely Event**: Same chance for different outcomes (e.g., heads or tails in a fair coin toss).
- **Unlikely Event**: Low chance of happening (e.g., winning a big lottery).
- **Impossible Event**: Will never happen (e.g., humans flying without wings).

Note: Probability helps us predict how likely something is to occur in real life!

Examples:

When we discuss real-Life scenario questions, we can find events and related examples:

Certain Event: We are in the sunny period. We are sure that the sun rises. If the sun rises tomorrow morning, this event is certain.

Likely Event: If you have a bag with 8 red marbles and 2 blue marbles, it is likely to pick a red marble.

Equally Likely Event: When tossing a fair coin, there are equal chances of getting a head or a tail. Therefore, getting a head is equally likely. If a bag has 5 green and 5 yellow balls, picking green or yellow is equally likely.

Unlikely Event: If you have a bowl with 95 black beads and 5 white beads, the chance of picking a white bead is very small. Therefore, it is unlikely to pick a white bead.

Impossible Event: A dog cannot give birth to a cat? Therefore, a dog giving birth to a cat is an impossible event.

Application activity 15.4

1. There are 4 corners in the class. Each corner has a cut out with a question. Work with your partner (in pairs) to visit each corner and work out the question you find there.

   Corner 1 (Weather Prediction): "If the weather forecast says there's a 90% chance of rain, is rain certain, likely, or unlikely?"

   Corner 2 (Sports): "If a football team has won all its matches this season, is it likely or unlikely they will win the next game?"

   Corner 3 (Games): "In a game with a spinner divided into 4 equal parts (red, blue, green, yellow), is landing on red equally likely as landing on blue?"

   Corner 4 (Imaginary Scenarios): "Is it impossible, unlikely, or certain that a tree can talk?"

2. Individual Task: write 3 real-life examples for each probability term (certain, likely, equally likely, unlikely, impossible).

15.5. End unit assessment

Assessing knowledge and understanding

1. Define:

a) What is an impossible event? Give one example.

b) Explain the term equally likely with an example.

2. Match:

Match each term to its correct definition:

Tem	Definition
1. Certain	A) High chance but not guaranteed
2. Unlikely	B) Will always happen.
3. Likely	C) Low chance but possible.

3. Identify:

Classify these events as certain, likely, equally likely, unlikely, or impossible:

a) Rolling a 7 on a standard die. _____

b) The sun rising tomorrow. _____

c) Picking a red marble from a bag with 1 red and 9 blue marbles. _____

Assessing skills

4. The table below shows the number of pupils of a school in each year

Year	2018	2019	2020	2021	2022	2023	2024	2025
Number of pupils	252	302	290	354	370	406	432	458

Use the language of chance to define these statements.

a) The school started in 2018 _____

b) The number of pupils will increase in the next year. _____

5. A card is drawn at random from a well schuffled deck of cards (52 cards). What is the likelihood of getting the king of diamond? (unlikely or more likely)?

6. There are 8 red pens, 3 blue pens and 1 black pen in a box. Complete the likelihood of the following event.

a) Picking a black pen: .....

b) Picking a blue pen: .....

c) Picking a green pen.....

d) Picking a black pen, a blue or a red pen.....

Assessing attitude and values

7. A lottery advert says, "You're likely to win!" Is this statement fair? Why or why not?
8. Your friend says, "It's equally likely to rain or not rain tomorrow." How would you verify this claim?
9. A football team won 8 out of 10 matches.
   a) Is it certain, likely, or unlikely they will win the next match?
   b) Should fans expect a win? Justify your answer.
10. Why is it important to understand probability in daily life? Give two examples.

REFERENCE

1. Rwanda Education Board (2015). Mathematics Syllabus for Upper Primary P4 - P6: Ministry of Education, Kigali.
2. Rwanda Basic Education Board (2020). Mathematics, Pupil's book, P6: Ministry of Education, Kigali.
3. Byamukama, J. & Mulisa, L. (2010). New Upper primary Maths Pupils Book Grade 6, Longman.
4. Male H., Kihara J., Mangale S. (2004). Understanding mathematics Primary 6, Longhorn Publishers, Nairobi.
5. Mugumu, D. & al. (2008). Mathematics Pupil's Book: Primary six. Kigali, Rwanda: MK 6. Publishers Ltd & NCDC.
6. Richard E (1997). Algebra I, Addison – Wesley Publishing Company, Inc, Phillipines.
7. Engelsohn H., Feit J (1980). Basic Mathematics, Moriah Publishing, New York.
8. Allen R (2004). Intermediate Algebra for College Students, Pearson Education, Inc, New Jersey.